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Multi Attribute Decision Making Based on Fuzzy Logic and Its Application in Supplier Selection Problem

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ABSTRACT

It is a complex situation when a decision is to be made under uncertainty. The uncertain information has been treated mathematically in the literature from different angles. This paper presents a novel method towards this problem based on fuzzy sets. The approach is constituted using distance between triangular fuzzy numbers. The proposed approach allows decision makers (DMs) to evaluate and improve supplier selection decisions in an uncertain situation. Finally, a numerical example on the supplier selection problem is proposed to illustrate an application of the methodology.

Keywords: *multi attribute decision making, supplier selection problem, triangular fuzzy numbers*

1. INTRODUCTION

Multi-attribute decision making is a very important task in management and decision sciences. Non-complex situations in this field include the conditions when the remarks of the Decision Makers (DM) on the attributes of consideration are completely known. This type of problems is not of our interest in this paper. We rather look for the situations where the DMs' various types of views are linguistic terms and thus not expressible by single real quantity. In this type of complex situation, the role of the DMs is very significant. Normally in all cases, a group of DMs is appointed to determine the choice among a set of finite number of alternatives. The tasks of the DMs are as follows:

i) to submit their opinions on the weights of the attributes,
ii) to submit their opinions on the alternatives for different attributes

Some authors have imposed an extra task in addition to these two. The philosophy behind their work is that the DMs are not equivalently important. So there needs a distinct way to compute the weights of the DMs. Keeping all these facts in mind several authors have tried to aggregate the outputs of the DMs in different ways. Kenny and Kirkwood (1975) have suggested the use of interpersonal comparison to obtain the values of scaling constraints in the weighed additive scale choice function. Bash (1980) has used a bargaining based approach to estimate the weights intrinsically. Mirkin (1979) has developed an eigen value method for deriving weightage of the DMs. In our proposed approach we exclude these concepts of previous methods on the hypothesis that the DMs may not be known to each other and so their mutual ranking may not enrich the decision making process.

Now coming to the point of the outputs a natural question arises: what is the type or form of the DMs' decision outputs for each attribute? Is it a single real quantity, or something else? If we run for the computational simplicity we will certainly seek out the single real quantity. But it may not fulfill our purpose, because the individual outputs of the DMs for individual attribute may vary a little with respect to time, which may influence the final decision. Thus for getting better result we should look for the linguistic terms as outputs, where unavoidably computational complexity occurs. Linguistic terms do not contain completely certain information. We can represent them in different ways like fuzzy triangular numbers, fuzzy trapezoidal numbers, grey interval numbers, gradual numbers, etc.

A number of quantitative techniques have been used for MADM problem by Triantaphyllou and Lin (1996), Kenny and Kirkwood (1975), Satty (1980), Liu and Liu (2010). Some of them are weighing method, statistical method, Analytical Hierarchy Process (AHP), data envelopment analysis, TOPSIS (Technique for Ordered Preference by Similarity to Ideal Solution) methods, etc. Research reveals that the application of AHP raises the decision making process and reduces the time taken to select the optimum alternative. TOPSIS assumes that each attribute has a tendency toward monotonically increasing or decreasing utility. So it is obvious that there exists one positive ideal and one negative ideal solution. The relative closeness of the alternatives is calculated and the ranking is obtained on the basis of that. Muralidharan et al. (2002) have used a novel model based on aggregation technique for combining DMs' preferences into one consensus ranking. Kumar et al. (2004) has used fuzzy goal programming towards this problem. Some approaches are also based on grey interval numbers, where input variables are considered as grey numbers (Li et al. (2007), Muley and Bajaj (2010)). There has been lot of researches on MADM by different authors also (Hwang and Yoon, 1981; Liu and Qiu, 1996; Bryson and Mobolurin, 1996; Da and Xu, 2002; Zhang and Fan, 2002; Wei et al., 2007; Wang, 2005; Liu, 2009; Liu and Liu, 2010).

Decision making for the selection of supplier from a given set has been acknowledged as a multicriteria problem consisting of both qualitative and quantitative factors. Several attempts have been made in order to solve different cases efficiently with the application of modern techniques like fuzzy or rough analysis.

In this paper we have presented an approach for solving MADM problems using operations on triangular fuzzy numbers (TFNs). In this approach it is assumed that the DMs are not equally important. The paper is structured as follows. Section 2 describes preliminary concepts on fuzzy sets and Triangular Fuzzy Numbers. Section 3 discusses our solution methodology. In section 4, the case study is presented to show the application of the methodology to a real industrial problem. Results of the application are finally presented and compared. Section 5 concludes the paper.

2. PRELIMINARIES

2.1 Fuzzy Sets

The concept of fuzzy logic was introduced by Zadeh (1965), when the two-valued logic completes its era. Initially it was given in prescribed form for engineering purposes and it got some time to accept this new methodology from different intellectuals. For a long time a lot of western scientists has been apathetic to use fuzzy logic because of it's threatening to the integrity of older scientific thoughts. But once it got the stage, it performed fabulously. From mathematical aspects to engineering systems, it spreaded its fragrances and the betterments of all types of systems were certainly there. After all, the society chose Fuzzy Logic as a better choice. In Japan, the first sub-way system was built by the use of fuzzy logic controllers in 1987. Since then almost every intelligent machine works with fuzzy logic based technology inside them. Apart from the engineering applications, Fuzzy sets and Fuzzy logic have been a handy tool for management and decision sciences. In this section we first submit some definitions on fuzzy sets.

Let us have a quick view of the following definitions.

Definition 2.1.1: Let X is a collection of objects called the universe of discourse. A fuzzy set denoted by $\tilde{A}$ on X is the set of ordered pairs $\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) : x \in X\}$ where $\mu_{\tilde{A}}(x)$ is the grade of membership of x in $\tilde{A}$ and the function $\mu_{\tilde{A}}(x) : X \rightarrow [0, 1]$ is called the membership function.

Definition 2.1.2: The support of a fuzzy set $\tilde{A}$ on X denoted by supp ($\tilde{A}$) is the set of points in X at which $\mu_{\tilde{A}}(x)$ is positive, i.e., supp ($\tilde{A}$) = $\{ x \in X : \mu_{\tilde{A}}(x) > 0\}$.

The core of a fuzzy set $\tilde{A}$ on X denoted by core ($\tilde{A}$) is the set of points in X at which $\mu_{\tilde{A}}(x)$ equals 1, i.e., core ($\tilde{A}$) = $\{ x \in X : \mu_{\tilde{A}}(x) = 1\}$.

The height of a fuzzy set $\tilde{A}$ on X denoted by height ($\tilde{A}$) is defined by height ($\tilde{A}$) = sup $\mu_{\tilde{A}}(x)$.

2.2 Fuzzy Numbers and TFNs

Definition 2.2.1 Let a be a given crisp number on the real line R. If there lies some uncertainty while defining a then we can represent a alongwith its uncertainty by an ordinary fuzzy number $\tilde{A}$. To represent $\tilde{A}$ mathematically and graphically a membership function $\mu_{\tilde{A}}(x)$ is used which must satisfy the following conditions:

1. $\mu_{\tilde{A}}(x)$ is upper semi continuous
2. In a certain interval [a, b] on R, $\mu_{\tilde{A}}(x)$ is non-zero, and otherwise it is zero.
3. There exists an interval [c, d] ⊂ [a, b] such that
   i) $\mu_{\tilde{A}}(x)$ is increasing in [a, c]
   ii) $\mu_{\tilde{A}}(x)$ is decreasing in [d, b], and
   iii) $\mu_{\tilde{A}}(x) = 1$ in [c, d].

Now a triangular fuzzy number (TFN) $\tilde{A}$ satisfies all the above conditions and it is represented by $\tilde{A} = (a, b, c)$.

Let us consider two TFNs $\tilde{X} = (x_1, x_2, x_3)$, $\tilde{Y} = (y_1, y_2, y_3)$ and a crisp number c. Then the basic arithmetic operations are as follows:

$\tilde{X} \oplus \tilde{Y} = (x_1 + y_1, x_2 + y_2, x_3 + y_3)$,

$\tilde{X} \ominus \tilde{Y} = (x_1 - y_1, x_2 - y_2, x_3 - y_3)$,

$\tilde{X} \otimes \tilde{Y} \approx (x_1 y_1, x_2 y_2, x_3 y_3)$ [Multiplication results an approximate fuzzy number] and $\tilde{X} \otimes c = (cx_1, cx_2, cx_3)$.

Definition 2.2.2 The distance between the TFNs $\tilde{X} = (x_1, x_2, x_3)$ and $\tilde{Y} = (y_1, y_2, y_3)$ is defined as (Chen (2000)):

$$d\left(\tilde{X}, \tilde{Y}\right) = \sqrt{\frac{1}{3}\left[\left(x_1 - y_1\right)^2 + (x_2 - y_2)^2 + (x_3 - y_3)^2\right]}$$

3. PROPOSED APPROACH

A new approach based on fuzzy distance is proposed for ordering the performance of alternatives.

Let us start with a multi attribute decision making problem in an uncertain environment where the set of n alternatives is X= $\{X_1, X_2, \ldots, X_n\}$, the set of m attributes is C= $\{C_1, C_2, \ldots, C_m\}$ and the set of p decision makers (DM) is D= $\{D_1, D_2, \ldots, D_p\}$. The alternatives are rated by p DMs on the basis of m attributes on the linguistic scale {Very Poor (VP), Poor (P), Fair (F), Medium Good (MG), Good (G) and Very Good (VG)}. Since the judgment is simply resembled by the linguistic terms, the vagueness of these terms assumed to be represented as triangular fuzzy numbers (TFN). The list of such TFNs is represented in Table 1. Also the m attributes

Table 1. Expression of linguistic terms in TFN

Linguistic Term for Attribute Ratings	TFN	Linguistic Term for Attribute Weights	TFN
Very Poor	(1, 2, 3)	Very Low	(0.1, 0.2, 0.3)
Poor	(2, 3, 4)	Low	(0.2, 0.3, 0.4)
Fair	(4, 5, 6)	Medium	(0.4, 0.5, 0.6)
Medium Good	(6, 7, 8)	Medium High	(0.6, 0.7, 0.8)
Good	(8, 9, 10)	High	(0.8, 0.9, 1.0)
Very Good	(9, 10, 10)	Very High	(0.9, 1.0, 1.0)

are rated by the DMs on the linguistic terms scale {Very Low (VL), Low (L), Medium (M), Medium High (MH), High (H) and Very High (VH)}. The corresponding TFNs are also represented in Table 1. The responses of the DMs are recorded in Table 2 and 3. Table 2 is the decision matrix on the rating of the alternatives with respect to the attributes, whereas Table 3 is the decision matrix on the rating of the attributes.

Step 1

The weights of importance of the decision makers are evaluated in terms of crisp nature. Table 2 is used here for the process. We consider the general element (which is surely to be a linguistic term from the Table 1) of the Table 2 as x_{ijk}, which is the decision of the kth DM of the ith alternative for the jth attribute. So $1 \le i \le n$, $1 \le j \le m$ and $1 \le k \le p$. We denote the TFN of x_{ijk} by TFN (x_{ijk}) = ($a_{ijk}, b_{ijk}, c_{ijk}$). For each attribute C_j, the TFNs corresponding to each column of Table 3.2 are averaged. The resulting TFN for the jth attribute and ith alternative is denoted by M_{ij} and we call it Mean. Now the weights of the DMs are evaluated from the following conceptual background. A Decision Maker gets higher weight if total deviation of his distinct decisions from the corresponding Means (Mathematical Mean of the proposed linguistic terms) is lesser. Actually here deviation means the distance between two TFNs, the TFN corresponding to the linguistic term of the Decision and the TFN corresponding to the mean of the submitted decisions. The distance formula is stated in section 2. The total deviation of the kth DM D_k is denoted by TD_k where

$$\mathrm{TD_k} = \sum_{j=1}^{m}\left(\sum_{i=1}^{n} d\left[\mathrm{M_{ij}}, TFN(\mathrm{x_{ijk}})\right]\right).$$

The total deviations thus obtained are normalized by their sum $\sum_{k=1}^{p} TD_K$ and the weight of D_k is defined as

$$\omega_{\mathrm{D_k}} = \frac{1 - \dfrac{\mathrm{TD_k}}{\sum_{k=1}^{p} TD_K}}{\sum_{k=1}^{p}\left(1 - \dfrac{\mathrm{TD_k}}{\sum_{k=1}^{p} TD_K}\right)} \quad \ldots (1)$$

Step 2

Now the job is to calculate the weights of the attributes in terms of TFNs using Table 3. In Table 3 the general element (which is also a linguistic variable) is considered as y_{jk} which is the decision of $\mathrm{D_k}$ on the rating of the attribute C_j and TFN (y_{jk}) $= \left(a_{jk}, b_{jk}, c_{jk}\right)$. The weight of the jth attribute is denoted as $\omega_{\mathrm{C_j}} = \left(a_j, b_j, c_j\right)$ and is defined by

$$a_j = \underset{k}{Min}(a_{jk} \times \omega_{\mathrm{D_k}}),$$

$$b_j = \frac{1}{p}\sum_{k=1}^{p} b_{jk} \times \omega_{\mathrm{D_k}} \text{ and}$$

$$c_j = \underset{k}{Max}(c_{jk} \times \omega_{\mathrm{D_k}})$$

Step 3

The TFNs of the elements x_{ijk} are multiplied by the corresponding DMs' weights and a new set of TFNs is obtained. Let us denote these TFNs by $\tilde{z}_{ijk} = \left(a_{ijk} \times \omega_{\mathrm{D_k}}, b_{ijk} \times \omega_{\mathrm{D_k}}, c_{ijk} \times \omega_{\mathrm{D_k}}\right)$.

Step 4

Now the fuzzy decision matrix (FDM) is obtained by aggregating the TFNs $\tilde{z}_{ijk}$. Consider the general element of the FDM as $\tilde{f}_{ij} = \left(p_{ij}, q_{ij}, r_{ij}\right)$. We define

$$p_{ij} = \underset{k}{Min}(a_{ijk} \times \omega_{\mathrm{D_k}}),$$

$$q_{ij} = \frac{1}{p}\sum_{k=1}^{p} b_{ijk} \times \omega_{\mathrm{D_k}} \text{ , and}$$

$$r_{ij} = \underset{k}{Max}(c_{ijk} \times \omega_{\mathrm{D_k}}).$$

Step 5

The elements of the FDM are now normalized by the greatest element $\underset{i,j}{Max}\left\{p_{ij}, q_{ij}, r_{ij}\right\}$ to restrict them to lie between 0 and 1.

Step 6

Now to construct the weighted normalized fuzzy decision matrix (WNFDM), each TFN of the NFDM is multiplied by its corresponding attribute's weight. Denote the general element of WNFDM by $\tilde{t}_{ij} = \left(l_{ij}, m_{ij}, n_{ij}\right)$.

Step 7

For each attribute C_j, a pseudo alternative is constructed and we call it 'Ideal Alternative' (IA). The TFN of IA for the *j*th attribute is denoted by $\left(l_j^{IA}, m_j^{IA}, n_j^{IA}\right)$ and defined by

$$l_j^{IA} = \underset{i}{Max}\left(l_{ij}\right)$$

$$m_j^{IA} = \underset{i}{Max}\left(m_{ij}\right) \text{ and}$$

$$n_j^{IA} = \underset{i}{Max}\left(n_{ij}\right)$$

The distance of each TFN of the WNFDM from the TFN of the corresponding IA is calculated and we denote it by $d_{ij}\left[\tilde{t}_{ij}, TFN\left(IA\right)\right]$. Also define

$$d_i = \sum_{j=1}^{m} d_{ij}\left[\tilde{t}_{ij}, TFN\left(IA\right)\right] \ldots (2),$$

which is the sum of the distances of *i*th alternative from the corresponding IA with respect to all attributes.

When $d_i < d_j$, *j*th alternative is more close to the IA than the *i*th alternative and gets higher rank. Once the rank is obtained, the process of decision making is complete.

Table 2. Decision matrix on the rating of suppliers

Attribute	Supplier	Decision Maker			
		D_1	D_2	…	D_p
C_1	X_1				
	X_2				
	X_3				
	…				
	Xn				
C_2	X_1				
	X_2				
	X_3				
	…				
	Xn				
…	X_1				
	X_2				
	X_3				
	…				
	Xn				
C_m	X_1				
	X_2				
	X_3				
	…				
	Xn				

Table 3. Decision matrix on the rating of attributes

	C1	C2	…	Cm
D1				
D2				
…				
Dp				

4. CASE STUDY AND ANALYSIS

In this section, the approach proposed is applied to a real case study, for a typical supplier selection problem.

Datre Corporation Limited is one of the largest Integrated Special Steel and Alloy Steel casting and precision valve manufacturing companies in Eastern India. Mainly the products of this reputed company are Specialized castings and high precision valves for the oil and gas fertilizers, Chemical, Petrochemical, Power and Steel Industries as well as Refineries. Idler (Drawing No TB00204) is one of the best selling products of this company whose MOC (Material of Construction) code is PH11. Idler is a part of Telecon Excavators. For this particular product the company needs the essential raw material Molybdenum (Mo). Five suppliers have been chosen by the company for this material. They are Lalwani Industries Limited, Rama Ferro Alloys, Amar Trade Limited, Minmat Ferro Alloys and Kothari Metals. All of them are from India. The company needs the best supplier that suits well with the four attributes: Product Quality (C_1), Service Quality (C_2), Delivery Time (C_3) and Price (C_4). A group of Decision Makers (D_1, D_2, D_3 and D_4) has been appointed by the company and they have been asked to submit their decisions on the rating of the suppliers as well as on the rating of the attributes. Here the 'decisions' are surely to be linguistic terms that are shown in Table1. In this case study, X_1, X_2, X_3, X_4 and X_5 are the five suppliers stated above. The DMs are eminent professionals and Experts in the concerned field. The 'decisions' of the DMs on the rating of the suppliers are shown in Table 4 whereas the 'decisions' on the rating of the attributes are shown in Table 5.

Table 4. Decision matrix on the rating of suppliers

Attribute	Supplier	Decision Maker			
		D_1	D_2	D_3	D_4
	X_1	VG	VG	G	G
	X_2	G	VG	VG	MG
	X_3	F	F	MG	F
C_1	X_4	P	P	F	VP
	X_5	F	F	F	VP
	X_1	VG	G	G	MG
	X_2	G	G	G	G
C_2	X_3	F	MG	MG	F
	X_4	F	MG	F	F
	X_5	G	MG	MG	G
	X_1	F	MG	F	F
	X_2	VG	G	G	G
C_3	X_3	G	VG	G	G
	X_4	F	F	MG	F
	X_5	G	G	G	G
	X_1	F	MG	MG	P
	X_2	P	F	F	P
C_4	X_3	G	G	G	MG
	X_4	F	F	MG	MG
	X_5	MG	G	G	MG

Table 5. Decision matrix on the rating of attributes

Attribute	Decision Maker			
	D_1	D_2	D_3	D_4
C_1	MH	MH	H	VH
C_2	H	H	MH	H
C_3	H	MH	H	H
C_4	VH	H	MH	VH

Step 1

The mean TFN of each row of the decision matrix is calculated. The distances of the DMs' TFNs from the corresponding mean are calculated and the total distance of each DM is evaluated. These total distances are substituted in equation (1) to evaluate the weights of the DMs in terms of crisp values. All these numeric are displayed in Table 6.

Step 2

The weights ω_{C_j} of the attributes C_j are calculated and displayed in Table 7.

Step 3

The TFNs $\tilde{z}_{ijk} = \left(a_{ijk} \times \omega_{D_k}, b_{ijk} \times \omega_{D_k}, c_{ijk} \times \omega_{D_k}\right)$ are evaluated and presented in Table 8.

Step 4

The FDM is constituted with the general element $\tilde{f}_{ij}$ and displayed in Table 9.

Step 5

The maximum entry in FDM is $\underset{i,j}{Max}\{p_{ij}, q_{ij}, r_{ij}\}$ = 2.670. All the elements of FDM are normalized by this element to construct the normalized fuzzy decision matrix (NFDM), displayed in Table 10.

Step 6

The elements of the NFDM are multiplied by the corresponding attribute weights and we obtain the weighted normalized fuzzy decision matrix WNFDM, shown in Table 11.

Step 7

The TFN for the Ideal Supplier, $\left(l_j^{IA}, m_j^{IA}, n_j^{IA}\right)$ is created by

$l_j^{IA} = \underset{i}{Max}\left(l_{ij}\right)$

$m_j^{IA} = \underset{i}{Max}\left(m_{ij}\right)$ and

$n_j^{IA} = \underset{i}{Max}\left(n_{ij}\right)$, and we obtain TFN (IS) = {(0.065, 0.145, 0.214), (0.097, 0.169, 0.267), (0.097, 0.173, 0.267), (0.102, 0.189, 0.267)}.

Table 6. Distances of the DMs from the corresponding means and weights of the DMs

	C_1	C_2	C_3	C_4	Total	Weights
D_1	2.262	3.561	1.816	3.704	11.343	0.267
D_2	2.969	4.533	2.816	4.704	15.022	0.245
D_3	3.262	2.854	2.408	4.704	13.228	0.256
D_4	5.579	4.171	1.408	6.112	17.270	0.232

Table 7. Weights of the attributes

	C_1	C_2	C_3	C_4
weights	(0.093, 0.163, 0.214)	(0.139, 0.200, 0.267)	(0.139, 0.200, 0.267)	(0.196, 0.237, 0.267)

Table 8. The TFNs $\tilde{z}_{ijk}$ on suppliers X_i's

Attribute	Supplier	Decision Maker			
		D_1	D_2	D_3	D_4
C_1	X_1	(2.403, 2.670, 2.670)	(2.205, 2.450, 2.450)	(2.048, 2.304, 2.560)	(1.856, 2.088, 2.320)
	X_2	(2.136, 2.403, 2.670)	(2.205, 2.450, 2.450)	(2.304, 2.560, 2.560)	(1.392, 1.624, 1.856)
	X_3	(1.068, 1.335, 1.602)	(0.980, 1.225, 1.470)	(1.536, 1.792, 2.048)	(0.928, 1.160, 1.392)
	X_4	(0.534, 0.801, 1.068)	(0.490, 0.735, 0.980)	(0.512, 0.768, 1.024)	(0.232, 0.464, 0.696)
	X_5	(1.068, 1.335, 1.602)	(0.980, 1.225, 1.470)	(1.024, 1.280, 1.536)	(0.232, 0.464, 0.696)
C_2	X_1	(2.403, 2.670, 2.670)	(1.960, 2.205, 2.450)	(2.048, 2.304, 2.560)	(1.392, 1.624, 1.856)
	X_2	(2.136, 2.403, 2.670)	(1.960, 2.205, 2.450)	(2.048, 2.304, 2.560)	(1.856, 2.088, 2.320)
	X_3	(1.068, 1.335, 1.602)	(1.470, 1.715, 1.960)	(1.536, 1.792, 2.048)	(0.928, 1.160, 1.392)
	X_4	(1.068, 1.335, 1.602)	(1.470, 1.715, 1.960)	(1.024, 1.280, 1.536)	(0.928, 1.160, 1.392)
	X_5	(2.136, 2.403, 2.670)	(1.470, 1.715, 1.960)	(1.536, 1.792, 2.048)	(1.856, 2.088, 2.320)
C_3	X_1	(1.068, 1.335, 1.602)	(1.470, 1.715, 1.960)	(1.024, 1.280, 1.536)	(0.928, 1.160, 1.392)
	X_2	(2.403, 2.670, 2.670)	(1.960, 2.205, 2.450)	(2.048, 2.304, 2.560)	(1.856, 2.088, 2.320)
	X_3	(2.136, 2.403, 2.670)	(2.205, 2.450, 2.450)	(2.048, 2.304, 2.560)	(1.856, 2.088, 2.320)
	X_4	(1.068, 1.335, 1.602)	(0.980, 1.225, 1.470)	(1.536, 1.792, 2.048)	(0.928, 1.160, 1.392)
	X_5	(2.136, 2.403, 2.670)	(1.960, 2.205, 2.450)	(2.048, 2.304, 2.560)	(1.856, 2.088, 2.320)
C_4	X_1	(1.068, 1.335, 1.602)	(1.470, 1.715, 1.960)	(1.536, 1.792, 2.048)	(0.464, 0.696, 0.928)
	X_2	(0.534, 0.801, 1.068)	(0.980, 1.225, 1.470)	(1.024, 1.280, 1.536)	(0.464, 0.696, 0.928)
	X_3	(2.136, 2.403, 2.670)	(1.960, 2.205, 2.450)	(2.048, 2.304, 2.560)	(1.392, 1.624, 1.856)
	X_4	(1.068, 1.335, 1.602)	(0.980, 1.225, 1.470)	(1.536, 1.792, 2.048)	(1.392, 1.624, 1.856)
	X_5	(1.602, 1.869, 2.136)	(1.960, 2.205, 2.450)	(2.048, 2.304, 2.560)	(1.392, 1.624, 1.856)

Table 9. Fuzzy decision matrix (FDM)

	C_1	C_2	C_3	C_4
X_1	(1.856, 2.378, 2.670)	(1.392, 2.201, 2.670)	(0.928, 1.372, 1.960)	(0.464, 1.384, 2.048)
X_2	(1.392, 2.259, 2.670)	(1.856, 2.250, 2.670)	(1.856, 2.317, 2.670)	(0.464, 1.000, 1.536)
X_3	(0.928, 1.378, 2.048)	(0.928, 1.150, 2.048)	(1.856, 2.311, 2.670)	(1.392, 2.134, 2.670)
X_4	(0.232, 0.692, 1.068)	(0.928, 1.372, 1.960)	(0.928, 1.378, 2.048)	(0.980, 1.494, 2.048)
X_5	(0.232, 1.076, 1.602)	(1.470, 2.000, 2.670)	(1.856, 2.250, 2.670)	(1.392, 2.000, 2.560)
weights	(0.093, 0.163, 0.214)	(0.139, 0.200, 0.267)	(0.139, 0.200, 0.267)	(0.196, 0.237, 0.267)

Table 10. Normalized fuzzy decision matrix (NFDM)

	C_1	C_2	C_3	C_4
X_1	(0.695, 0.891, 1.000)	(0.521, 0.824, 1.000)	(0.348, 0.514, 0.734)	(0.174, 0.518, 0.767)
X_2	(0.521, 0.846, 1.000)	(0.695, 0.843, 1.000)	(0.695, 0.868, 1.000)	(0.174, 0.374, 0.575)
X_3	(0.348, 0.516, 0.767)	(0.348, 0.562, 0.767)	(0.695, 0.866, 1.000)	(0.521, 0.799, 1.000)
X_4	(0.089, 0.259, 0.400)	(0.348, 0.514, 0.734)	(0.348, 0.516, 0.767)	(0.367, 0.560, 0.767)
X_5	(0.089, 0.403, 0.600)	(0.550, 0.749, 1.000)	(0.695, 0.843, 1.000)	(0.521, 0.749, 0.959)
weights	(0.093, 0.163, 0.214)	(0.139, 0.200, 0.267)	(0.139, 0.200, 0.267)	(0.196, 0.237, 0.267)

Table 11. Weighted normalized fuzzy decision matrix (WNFDM)

	C_1	C_2	C_3	C_4
X_1	(0.065, 0.145, 0.214)	(0.072, 0.165, 0.267)	(0.024, 0.103, 0.196)	(0.034, 0.123, 0.205)
X_2	(0.048, 0.138, 0.214)	(0.097, 0.169, 0.267)	(0.097, 0.173, 0.267)	(0.034, 0.089, 0.154)
X_3	(0.032, 0.084, 0.164)	(0.048, 0.112, 0.205)	(0.097, 0.173, 0.267)	(0.102, 0.189, 0.267)
X_4	(0.008, 0.042, 0.086)	(0.048, 0.103, 0.196)	(0.048, 0.103, 0.205)	(0.072, 0.133, 0.205)
X_5	(0.008, 0.066, 0.128)	(0.076, 0.150, 0.267)	(0.097, 0.169, 0.267)	(0.102, 0.178, 0.256)

Table 12. Individual distances and total distance between t_{ij} and IA

	d_{i1}	d_{i2}	d_{i3}	d_{i4}	Total
X_1	0	0.015	0.071	0.065	0.151
X_2	0.01	0	0	0.096	0.107
X_3	0.036	0.056	0	0	0.092
X_4	0.100	0.063	0.061	0.051	0.275
X_5	0.075	0.016	0.002	0.009	0.102

The distances of the suppliers from the Ideal Supplier, d_k are calculated using equation (2). The results are displayed in Table 12. According to the result, we conclude that $X_3 > X_5 > X_2 > X_1 > X_4$.

At the end of our discussion we present a comparative analysis of several supplier selection methods and indicate some advantages of using our proposed methodology based on Triangular Fuzzy Numbers. There are huge numbers of approaches towards this problem. We collect only some of those (Chen' 2000, Li, Yamaguchi and Nagai' 2007, Sreekumar and S. S. Mahapatra' 2009) where uncertain information has been used. In each of these methods linguistic terms have been used for representing DMs' decisions on the suppliers with respect to the attributes.

Table 13 shows the comparison based on different techniques used to obtain the solution. This is clear from Table 13 that apart from the approach of Sreekumar and Mahapatra, the weights of the DMs are nowhere considered. The justification of considering DMs' weights has already been discussed in section 1. In Sreekumar and Mahapatra's method (2009), the authors evaluated DMs' weights using Eigen Vector method, proposed by Mirkin (1979). In this method, each DM is asked to rank the others on a certain scale. The judgment matrix is thus obtained and the normalized Eigen Vector stands for the DMs' weights. Now this methodology suffers from the following facts:

1. The Decision Makers may not know each other, may know something about each other. So the ranking procedure may not be truthful in those cases.
2. If a Decision Maker is biased to any particular supplier, then the biasness stays and effects in the whole evaluation process very strongly.

In our proposed approach, the above mentioned drawbacks are not present, as:

1. No mutual ranking is present here. The weights are evaluated by calculating total deviation from the mean of the decisions.
2. Biasness, if present, is reduced normally in this method.

In view of the above discussion we certainly ensure that the proposed approach delivers better judgment in all conditions than the others. Now we apply our method to the grey based approach, proposed by Li et al. (2007). The DMs are supposed to be equally important there. But applying our method, the weights are found to be as represented in Table 14.The distances are calculated on the basis of the decisions on supplier S_1. Considering these weights the ranking of the six suppliers is obtained as $S_1>S_2>S_4>S_5>S_6>S_3$ which is a little bit different $S_1>S_2>S_4>S_5>S_3>S_6$ as evaluated in Li et al.'s method (2007).

Table 13. Comparison with other methods

Approaches	Way of Expressing linguistic terms	Whether DMs' weights are considered	Whether compared to the ideal supplier	Scale of representation
Chen (2000)	Triangular Fuzzy Number	No	Yes	7 point
Li, Yamaguchi and Nagai (2007)	Grey Numbers	No	Yes	7 point
Sreekumar and S. S. Mahapatra (2009)	Triangular Fuzzy Number	Yes	Yes	6 point
Proposed Method	Triangular Fuzzy Number	Yes	Yes	6 point

Table 14. Distances from the corresponding means and weights of the DMs

D_i	C_1	C_2	C_3	C_4	Total	Weights
D_1	2.6514	7.0232	10.7208	38.5650	58.9604	0.21
D_2	22.8650	10.7070	10.6542	5.3034	49.5296	0.23
D_3	2.6520	9.9870	10.0785	5.3034	28.0209	0.28
D_4	2.6510	10.6780	9.7864	5.3450	28.4604	0.28

5. CONCLUSION

MADM problem is of great importance for its various application fields. The supplier selection problem is a complex decision making problem that includes both qualitative and quantitative factors, which are often assumed using imprecise data and human judgments. It then therefore appears that fuzzy MADM is well suited to deal with such decision making problems. In most of the decision making environments, DMs' assessments are often uncertain and should not be counted on exact numeric values. Thus supplier selection problem also deals with uncertainties, which has been mathematically treated by triangular fuzzy numbers, in this paper. The distance formula has been borrowed from Chen (2000). In this current approach, a novel technique has been implemented to extract DMs' weights from the decision matrix to obtain more reliable decision. The methodology has been exemplified by a real case study, taken from supplier selection problem and the comparative analysis clarified the effective impact of the approach. As an extension of the current methodology, the same algorithm for extracting DMs' weights can be applied to grey based techniques towards solving supplier selection problems. On the other hand more research on DMs' weights may reveal more reliable solutions.

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