Optimization of Inventory for Two Level Multiple Retailers-Single Manufacturer Reverse Supply Chain

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ABSTRACT
This paper deals with modelling of deteriorating used products inventory in a two level reverse supply chain. A time dependent optimisation model is developed for multi-retailer single manufacturer reverse supply chain involving collection and disposal of used products from customers. Collection rate or supply is assumed to exponential with time and time dependent deterioration of items is considered. Two cases are presented with one on decentralised policy optimizing retailers and manufacturer’s cost separately and another on centralised policy optimizing system wide cost. The optimal collection and disposal cycle times are derived and discussed with numerical example. The model has potential to be used in industries where existing distribution system is used for end-of-life products collection and disposal.

Keywords: time dependent deterioration, reverse supply chain, deterministic supply, optimal cycle time, economic order quantity

1. INTRODUCTION
A supply chain is a complex system consisting of many entities like supplier, manufacturer and retailer to deliver product(s) or service(s) to customers. Managing inventory across supply chain network is a complex problem. The inventory issues are related with cycle inventory, safety inventory, product availability, fill rate, order fill rate, cycle service level, and joint replenishment policies. Similar to forward supply chain, the used products or end-of-life (EOL) products from customers after its useful life are flowing back in reverse direction in the supply chain network. This reverse flow takes place in some of the industrial supply chain networks like automotive batteries and mobile phones.

The reverse supply chain is one of the emerging areas of research in the field of supply chain management. The reverse supply chain consists of list of activities involved in recovering EOL products for the purpose of recycling, remanufacturing and proper disposal by a manufacturer. Guide and Wassenhove (2002) defined Reverse supply chain or reverse logistics as a series of activities required to retrieve an EOL product from a customer and dispose of it properly or reuse after processing. The chain connects end users with manufacturer in reverse direction. Reuse and remanufacturing of products and materials are not new in the industry. Wastepaper recycling, metal scrap brokers and soft drink industry are all examples that have been around for a long time. In these cases, recovery of the used products is economically more attractive than disposal. But, in the recent past, there is a growing interest in reuse and remanufacturing among industries due to various reasons and that lead to more research studies in the area of reverse logistics.

Some of the used products collected from the market have significant commercial value and organisations are willing to recover that value. Many organisations in electronic industry and other organisations that handle high value but short life cycle products are more inclined to collect used products, and recover maximum value from them. Many organisations use their existing distribution channel partners to recover used products. The inventory management of used products after collection at channel partner level and also at re-manufacturing or disposal level is gaining importance due to following factors:
(i) The EOL product recovered may deteriorate in value
(ii) Costs of holding and transportation are significant

Inventory of deteriorating items first studied by Whitin (1957) and he considered the fashion goods which lose value with time. There were many studies related to deterministic mathematical models of inventory in single and multi-level supply chains. Most recent works on this are: Shibsankar and Chaudhuri(2003),Shah and Acharya (2008), Tripathy and Mishra (2010a), Tripathy and Mishra (2010b), Valliathal and Uthayakumar (2009), Begum et al., (2010), etc.

In a paper by Ouyang et al (2005), the authors proposed an EOQ inventory model for deteriorating items with exponential decreasing demand. This paper addresses shortages and partial backordering. Mandal et al (2007) provide an optimal batch production model which takes care of effect of continuous price decrease and time value of money on optimal decisions for inventoried goods having time dependent demand and production rates. The demand is assumed to follow exponential increase with time. Also the production rate also is assumed to follow exponential increase with time and in our work, it is assumed that collection rate or supply of EOL products increases exponentially with time.

A production inventory model for time dependent deteriorating items with production disruptions in which no shortages are allowed was developed by Mishra and Singh (2011). The model is used to determine optimal production time during normal and disrupted production periods. Shah et al., (2005) presented an EOQ model for deteriorating...
inventory items with temporary price discount. This model is trying to achieve balance between cost of ordering large quantity because of price discount and increase in cost of deterioration and cost of holding the inventory. The authors have assumed demand as constant per unit of time. In a review paper on deteriorating inventory, Goyal and Giri (2001) classified the relevant literature by the shelf-life characteristics of the inventoried goods. The sub-classification was done on the basis of demand variations and various other conditions and constraints. Berman et al. (2008) developed an EOQ model in which the content level is modelled by a Brownian Motion (BM) with state-dependent drift and diffusion. They derived all the relevant cost functional for the discounted case with infinite horizon and for the long-run average case. These explicit results are used for finding the optimal replenishment level in the long-run average case.

Vidovic et al., (2011), in their paper proposed a mathematical model to optimize recyclable collection process including inventory. Apart from optimizing transportation, recycling costs, the model propose optimization of inventory costs both at collection centres and at recycling facilities. In the work presented by Pattnaik (2012), an entropic order quantity model is presented for perishable items with two component demand in which the criterion is to optimise the expected total finite horizon payoff. This model represents an appropriate combination of two component demand with entropy cost, particularly over a finite time horizon. Its main aim lies in the need for an entropic cost of the cycle time is a key feature of specific perishable products like fruits, vegetables, food stuffs, fishes etc.

Chen and Chen (2005) studied the effects of joint replenishment and channel coordination for managing a multiple deteriorating products in a supply chain. The study assumes a linearly proportional demand for the items but with exponential deterioration. The result of their experiments also indicates that the savings are higher in the case of joint replenishment and channel co-ordination. Hajiet al. (2011) in their paper deals with a two-echelon inventory system consisting of one supplier and N retailers. Each retailer faces an independent Poisson demand with the same rate and applies a new ordering policy called one-for-one-period ordering policy for its inventory control. They obtained the optimal time interval between any two consecutive orders as well as the optimal average inventory for each retailer.

In an EOQ model for reverse logistics proposed by Teunter (2001), the author analysed a model with a return flow of items and the model had deterministic demand and return rates. He considered three different holding costs for manufactured items, returned items and remanufactured items. Gou et al., (2007) have presented a centralised transportation, inventory and handling control policy for an open-loop reverse supply chain. Using the method of stochastic process and queuing theory, the inventory model is formulated. A simulation is also used to search for optimal solution. In an EOQ model proposed by Chiu et al., (2009), the lead time factor, stochastic demand and returns were considered.

This paper is an attempt to investigate the inventory policies of EOL products in a reverse supply chain. In our case, reverse supply chain has one manufacturer and multiple retailers (Figure 1). The retailers collect EOL products from customers, while selling new products and the supply of EOL products is deterministic and exponential with time proportional. The retailer has to ship the collection quantity to manufacturer after significant inventory build-up. The manufacturer collects used products from all retailers and disposes or recycles them. The manufacturer is dominant partner because of extended producer responsibility which means manufacturer is legally responsible for proper collection and disposal of recycling of EOL products.

Based on the review, it is proposed to develop simple optimised inventory policies for two level reverse supply chains with deteriorating inventory. First a simple time dependent optimized model is developed for both retailers as well as for manufacturer independently (De-centralised policy-Case I). In the decentralised collection and disposal decision making policy, each entity within supply chain aims to minimize its own cost functions.

Then a coordinated centralised inventory policy using optimization model (Case II) also investigated. Here the inventory is managed jointly (Centralised policy) and it determines the shipping and disposal cycle by considering the total cost incurred by the retailer and the manufacturer, so that the system wide cost is minimized. The methodology of centralised and de-centralised inventory policies and optimisation method used in Chen and Chen (2005) study is taken as guidelines for the research. The paper is divided into three sub-sections. The following section explains the notations with assumptions. The second section proposes the model. The third section presents the numerical study and final section discusses the use of this model and provides directions for future research.

![Figure 1 Two Level Reverse Supply Chain](image)

**2. ASSUMPTIONS AND NOTATIONS**

Before presenting the notations used for the problem we list the main assumptions of the model:

(a) A two-stage supply chain consisting of a set of retailers and a manufacturer is considered.

(b) Single item with time dependent deterioration is collected by retailers and sent to manufacturer.

(c) Collection of returns or supply is deterministic and it increases exponentially with time. It is also assumed that used products are returned when new products are bought. Usually the sales of new items is more towards end of each period say a month due to sales pressure to achieve the target and hence the return volume also more towards the end of the period.

(d) There are two setup costs for manufacturer, one for inbound movement of retailer’s collection into...
manufacturer and another related to disposal of collected returns for further processing.

(e) Setup costs are assumed to fixed and related to transportation

(f) Holding cost per unit is fixed for both retailer and manufacturer

(g) The cycle time is common for both for collection and disposal in case II. The total disposal in a period is equal to total collection from all retailers in the previous period of same length. It is assumed that when old products returned by customers, the retailer collects and accumulates these over a period of time. The retailer has to incur holding cost for each period before the manufacturer collects them from the retailer. Ultimately, the retailer wants to ship the used products as early as possible to manufacturer. But, it is assumed that manufacturer takes responsibility of moving the used products from retailer, and hence he like to visit retailer at fixed intervals of time which is economical for him.

The Figure 2 explains the situation better. It is assumed that the manufacturer collects accumulated quantity from each retailer in one cycle and wants to recycle or dispose them in next cycle. The top two diagrams in Figure 2 describe how used products inventory is built over time in each cycle for two retailers. The bottom diagram in Figure 2 represents how used products inventory is declining with time after collection in the previous cycle from retailers. The used products are disposed for recycling and that represent the decline in inventory.

**Notations:**

- Ti – the individual shipment cycle of the retailer in case I, i=1,2, ….,n.
- Tm – the collection and disposal cycle of item for the manufacturer in case I.
- Ri – the setup cost per shipment from retailer to manufacturer.
- F – the setup cost (transportation for disposal) per lot for the manufacturer.
- Θ – the deterioration rate of item facing both the retailer and the manufacturer.
- Ai – initial collection of retailer.
- λi – is a constant governing the increasing rate of demand.
- hr,i, (hm) – the inventory holding cost of the inventory per unit for retailer and manufacturer.
- Ir,i(t), (Im (t) – the inventory level for retailer ‘i’ at time t (manufacturer, respectively).
- R – Total disposal volume in a cycle.
- TCr,i, (TCm) – total cost for the retailer (manufacturer respectively) per unit time under case I.
- TC* – total cost for the supply chain per unit time under case II.
- T* – optimum collection and disposal time for the manufacturer for the case II.

3. **MATHEMATICAL MODEL FORMULATION**

3.1 **Case I Inventory is Managed Independently**

In the decentralised collection and disposal decision making policy, each entity within supply chain aims to minimize its own cost functions. Each retailer makes a shipping decision based on policy that minimizes cost of holding as manufacturer bear the cost of transportation. The manufacturer also try to reduce its cost which consist of holding and shipment costs and includes additional transportation cost of moving inventory from each retailer to manufacturer warehouse.

3.1.1 **Retailer’s Policy**

If each retailer is allowed to dispose inventory independently then he wants to minimize his cost. The cost function is derived as follows:

The end-of-life products are collected by a retailer at the rate of:

\[
Ci(t) = Ai e^{λit}, \quad Ai > 0, \quad λi > 0
\]

So, the inventory at the retailer at any point of time with inventory deteriorates at ‘ Θ ’ rate is given by a differential equation,

\[
\frac{dIr,i(t)}{dt} = Ci(t) - Θ Ir,i(t) \quad \text{for } 0 \leq t \leq Ti
\]

Using the method proposed by Spiegel (1960), and the boundary condition of inventory,

\[
Ir,i(0) = 0
\]

\[
Ir,i(t) = \frac{Ai}{(0-λi)} [e^{λit} - e^{-Θt}]
\]

The amount of inventory accumulated for retailer ‘i’ over a period Ti is given by:

\[
\int_0^{Ti} Ir,i(t)dt = \frac{Ai}{(0-λi)} \left( e^{λiT_i} - 1 \right) / Θ + \left( e^{ΘTi - 1} - 1 / Θ \right)
\]

The total inventory holding cost is given as Hr,i:

\[
Hr,i = \frac{hr,i}{Ti} \int_0^{Ti} Ir,i(t) dt
\]

\[
= \frac{hr,i}{Ti} \frac{Ai}{λi + Θ} \left( e^{λiT_i} - 1 \right) / λi + \left( e^{ΘTi - 1} / Θ \right)
\]

Since the retailer incurs only this cost and transportation is taken care by manufacturer, he wants to reduce only this cost. The above equation is differentiated and equated to zero. Using the Taylor series expansion for
exponential terms and neglecting the third and higher order terms, the optimal shipment cycle time for retailer ‘i’ is given as:

\[ T_i = \frac{1}{(0-\lambda i)} \]  

(6)

### 3.1.2 Manufacturer’s Policy

The manufacturer collects used products from all retailers and uses these for recycling or disposal. He needs to dispose the entire inventory before the next arrival from retailers. The disposal or remanufacturing consumption rate is given as \( D(t) = \beta t \) for \( 0 \leq t \leq T_m \) and \( \beta > 0 \).

The total disposal over the disposal cycle is given by:

\[ R = \int_0^{T_m} D(t) \, dt \]

(7)

\[ R = \frac{\beta}{2} T_m^2 \]

(8)

So, \( \beta \) can be written as \( \beta = \frac{2R}{T_m^2} \).

The inventory level at the manufacturer at any point of time is given by the differential equation,

\[ \frac{dI_m(t)}{dt} = -D(t) - \lambda I_m(t) \quad \text{for} \quad 0 \leq t \leq T_m \]

(9)

The above equation can be re-written as:

\[ \frac{dI_m(t)}{dt} = -\frac{2R}{T_m^2} - \lambda I_m(t) \quad \text{for} \quad 0 \leq t \leq T_m \]

(10)

Using the method to solve differential equations as proposed by Spiegel (1960), and the boundary condition of inventory \( I_m(T_m) = 0 \).

\[ I_m(t) = \frac{2R}{\theta^2 T_m^2} \left[ (1 - \theta t) + (\theta t - 1) e^{\theta (T_m - t)} \right] \]

(11)

\[ \int_0^{T_m} I_m(t) \, dt = \frac{2R}{\theta^2 T_m^2} \left[ T_m - \frac{\theta T_m^2}{2} + \frac{(\theta - 1)(e^{\theta T_m} - 1)}{\theta} \right] \]

(12)

The above equation (3.12) represents the inventory accumulated over the period \( T_m \). For manufacturer, the cost of reverse inventory is as follows:

(i) Cost of transportation of collected quantity by each retailer to manufacturer

(ii) Cost of holding the collected inventory

(iii) Cost of disposal of this inventory

The total cost for the manufacturer is given as:

\[ TC_m = \sum_{i=1}^{n} \frac{R_i}{T_i} + \frac{F}{T_m} + \frac{h_m}{T_m} \left( \frac{2R}{\theta^2 T_m^2} \left[ T_m - \frac{\theta T_m^2}{2} + \frac{(\theta - 1)(e^{\theta T_m} - 1)}{\theta} \right] \right) \]

(13)

The first part of the equation refers to transport cost of used products from each retailer. The second part refers to fixed cost of transportation involved in disposal of inventory by the manufacturer. The third part refers to holding cost of inventory at manufacturer place. The above equation is differentiated with respect to \( T_m \) and equated to zero. Using the Taylor series expansion for exponential terms and neglecting the third and higher order terms, the optimal collection and shipment cycle time for manufacturer is given as:

\[ T_m^* = \frac{6}{\sqrt{\theta^2 - 2\lambda_1 R_i + F + \frac{2h_m}{T}} \left( \frac{2R}{\theta^2 T_m^2} \left[ T_m - \frac{\theta T_m^2}{2} + \frac{(\theta - 1)(e^{\theta T_m} - 1)}{\theta} \right] \right)} \]

(14)

The research question is “What are the supply chain management integration problems and possible solutions in an organisation?”

### 3.2 Case II Inventory is Managed Jointly (Centralised Policy)

Centralised policies determine the shipping and disposal cycle by considering the total cost incurred by the retailer and the manufacturer, so that the system wide cost is minimized. The total cost equation is given as a combination of equation (5) and equation(13) and replacing \( T_i \) and \( T_m \) with \( T \).

\[ TC = \frac{h_r}{T} \left( \frac{A_i}{\lambda_i + \theta} \left( e^{\lambda T} - 1 \right) / \lambda_i + \left( e^{\theta T} - 1 \right) / \theta \right) + \sum_{i=1}^{n} \frac{R_i}{T_m} + \frac{F}{T} + \frac{h_m}{T} \left( \frac{2R}{\theta^2 T^2} \left[ T - \frac{\theta T^2}{2} + \frac{(\theta - 1)(e^{\theta T} - 1)}{\theta} \right] \right) \]

(15)

The above equation is differentiated with respect to \( T \) and equated to zero. Using the Taylor series expansion for exponential terms and neglecting the third and higher order terms, the optimal collection and disposal cycle time for overall system is given as:

\[ T^* = \frac{12h_m R}{\left( \frac{12h_m R}{\theta^3} \sum_{i=1}^{n} h_r A_i \left( 1 + \frac{(\lambda_i - \theta)}{2} \right) - \sum_{i=1}^{n} R_i - \frac{F + 2h_m R}{\theta} \right)} \]

(16)

Since \( T^* \) remains on the right hand side of the equation substituting the value of \( T^* \), given by the equation above, results in:

\[ T^* = \sqrt{\frac{12h_m R}{\left( \frac{12h_m R}{\theta^3} \sum_{i=1}^{n} h_r A_i \left( 1 + \frac{(\lambda_i - \theta)}{2} \right) - \sum_{i=1}^{n} R_i - \frac{F + 2h_m R}{\theta} \right)} - \sum_{i=1}^{n} R_i - \frac{F + 2h_m R}{\theta}} \]

(17)

The term \( \left[ \sum_{i=1}^{n} h_r A_i \left( 1 + \frac{(\lambda_i - \theta)}{2} \right) \right] \) inside the square root in the denominator will have only a minor effect on the solution and may be ignored. So, the optimal collection and disposal cycle time for overall system is given by \( T^* \).

\[ T^* = \sqrt{\frac{12h_m R}{\left( \frac{12h_m R}{\theta^3} \sum_{i=1}^{n} h_r A_i \left( 1 + \frac{(\lambda_i - \theta)}{2} \right) - \sum_{i=1}^{n} R_i - \frac{F + 2h_m R}{\theta} \right)} - \sum_{i=1}^{n} R_i - \frac{F + 2h_m R}{\theta}} \]  

(18)
4. NUMERICAL STUDY

The inventory models that were developed can be applied in solving multi-retailer-single manufacturer problem with some numerical values. To illustrate the effect of the models, we have assumed three retailers and one manufacturer involved in collection and disposal of used products from customers. The parameter settings are given in Table 1.

The first analysis was conducted by changing the deterioration rates on both sides from the basic setting of 0.25. The cycle time of Tr, Tm and T are studied and compared with this variation and it is presented in Table 2 and Figure 3. From the graph, it is clear that when deterioration rate increases, each party want to reduce the cycle time and want to ship the product very fast so that maximum value can be recovered. For manufacturer, the difference in cycle time is not significant between case I and case II (Tm and T*). The T* was derived after adding holding cost of all retailers in the total cost function. Thus, the holding cost of retailers is not significant in determining the optimal collection and disposal cycle time but deterioration rate play an important role.

The second analysis was done to find out the variation of Tm and T* with respect to holding cost of manufacturer and also Fixed transportation cost. From the Table 3 it is interpreted that both Tm and T* are declining with respect to increase in holding cost of manufacturer and their values are almost similar. Also when fixed cost increases, the collection and disposal time also increases at smaller rate for lower values of fixed cost but it increases at an increasing rate when fixed cost increases beyond 2000 (Table 4).

<table>
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<tr>
<th>Table 1 Parameter Settings</th>
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<td><strong>Retailer</strong></td>
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<td>Retailer 3</td>
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<td><strong>Manufacturer</strong></td>
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<tr>
<th>Table 2 Collection and Disposal Cycle Time for Various Levels of Deterioration Rate</th>
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<th>Table 3 Cycle Time Against Holding Cost</th>
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<th>Table 4 Cycle Time Against Fixed Cost</th>
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5. CONCLUSIONS

This paper deals with a simple time dependent optimization model for multi-retailers-single manufacturer reverse supply chain involving collection and disposal of used products from customers. Collection rate or supply is assumed to be exponential with time and time dependent deterioration is considered. Both decentralised and centralised policies are studied with respect to reverse inventory. It is found that deterioration rate is significant in determining cycle time for both policies. It is also found that not much difference exist between centralised policy and decentralised policy with respect to manufacturer in terms of collection and disposal cycle time. The model has potential to be used in industries where existing distribution system sells new products after collecting end-of life products from customers. The possible likely industries which can use this model are batteries, Electronic items like Cell phones, Computers, Refrigerators.

One line of future research could study inventory models with a different distribution patterns in collection apart from exponential. A second line could extend current models and integrate with vehicle routing of collection with milk run among retailers. Finally, it would also be interesting to study reverse inventory systems with a stochastic supply pattern and deteriorating items.
REFERENCES


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