

Dynamics of Coordination for Return Policy Contracts with Warranty

Shirsendu Nandi

Department of Operations, Indian Institute of Management Rohtak, Haryana, India

Email: shir.05math@gmail.com

ABSTRACT

The purpose of this paper is to determine the relationship among different variables and contract parameters in order to achieve coordination for buyback contract and quantity flexibility contract with warranty. The paper analyses the dynamics of coordination and performs numerical analysis to compare the results obtained for different demand distributions. The paper makes use of analytical model and optimization techniques to investigate the dynamics of coordination. This study finds relationship among different exogenous variables and contract parameters to achieve channel coordination through warranty period optimization. The study also finds that with increase in mean of the distribution the optimal warranty length decreases. It provides the graphical nature of the risk and profit allocation for both the parties in the supply chain with increase in flexibility, buyback rate. It is found that in case of exponential demand distribution with higher variance, the manufacturer is required to offer a higher flexibility to the retailer in terms of quantity ordered by fixing a relatively larger flexibility parameter to ensure that both the parties in the supply chain have a positive profit. Using the demonstrated guidelines the coordinator of the supply chain may optimally design the contract parameters, warranty length etc. The study contributes to the existing literature by deriving necessary conditions for achieving supply chain coordination in case of a buyback contract and a quantity flexibility contract with warranty. The study helps the channel coordinator to understand the dynamics of coordination.

Keywords: buyback, quantity flexibility, supply chain coordination, warranty

1. INTRODUCTION

Businesses are failing to achieve competitive advantage due to supply chain integration silos (Msimangira and Venkatraman, 2014). Different supply chain contracts have been mentioned in literature as a means of achieving channel coordination. Buyback contract and quantity flexibility contract are categorized as return policy contracts. These contracts are practised widely in different industries as a mechanism for achieving optimal supply chain profit as well as increasing the profit level of both the parties (upstream and downstream) in the supply chain. Buyback contract is adopted in different industries such as high-tech, fashion apparel etc. (Donohue *et al.*, 2011). In a supply chain consisting of one manufacturer (supplier) and one retailer (buyer), the supplier is initially paid a flush of cash according to the number of units purchased and the wholesale price per unit of the product. At the end of the selling season the unsold items are returned back by the supplier at a pre specified buyback rate. Quantity flexibility contract is widely applied in industries like consumer electronics, computer particularly in companies such as IBM, HP, Compaq etc. (Cao *et al.*,

2009). This contract is useful because the retailer gets an opportunity to revise its ordering decision upto a certain level after having a close observation of the market demand. Therefore, this particular contract works on the basis of a predetermined fraction (α) such that at the end of the selling season at most αq number of unsold items are returned back by the manufacturer with full credit.

It is also evident that in case of the products associated with the aforesaid industries where these two return policy contracts are adopted, warranty decision plays a very vital role. Selling of the products related to high-tech or apparel industry, computer industry or electronics industry generally involves offering a warranty which acts as a marketing element to signal better reliability of the product. Decision regarding the length of the warranty becomes important since it has an impact on the demand and at the same time cost is associated with it. Offering a longer warranty period may result in huge cost towards fulfilling warranty obligations and may influence demand positively. On the contrary, a warranty period of relatively shorter length may reduce demand by negatively influencing customers' buying decision although this may potentially decrease the expected warranty cost. Therefore, choosing an appropriate warranty period is very important. So, the coordinator requires a guideline in terms of the relationship among different variables to decide the contract parameters and the optimal warranty length in an appropriate manner. Therefore, the channel coordination problem in conjunction with warranty with respect to the aforesaid two return policy contracts is relevant to many industries.

High-tech consumer electronics appliances, computers are becoming products with short life-cycle characterized by short selling season, uncertain demand. The market for these products has become volatile because of continuous innovation in technology and frequent changes in customer needs. Therefore, the pattern of demand observed varies across products related to these supply chains. The demand patterns may be approximated by uniform, normal and exponential distribution in the case of cyclic demand with trend, stochastic demand and lumpy demand respectively (Choy and Cheong, 2011). The challenge for the global coordinator is to design different contract parameters and set an appropriate warranty length in order to achieve channel coordination keeping in view the nature of demand. Limited literature is available related to supply chain contract in conjunction with warranty (Hu, 2008; Ji *et al.*, 2011; Sinha and Sarmah, 2011; Dai *et al.*, 2012; Lan *et al.*, 2014). However, there is no existing literature on the coordination mechanism with respect to buyback contract and quantity flexibility contract in conjunction with warranty. How the dynamics of coordination varies with different demand

distributions lacks attention of the researchers and needs to be explored. It is also important to understand the impact of change in the exogenous parameters like cost of production and average warranty cost, retail price, parameters of the warranty dependent demand function on the optimal order quantity, optimal warranty length, supply chain profit and supply chain risk. Therefore, the present study aims at addressing the aforesaid issues.

The rest of the current paper is organized in the following manner. Section 2 deals with the detailed literature review of relevant literature. Section 3 states the necessary assumptions and builds the analytical model. Section 4 narrates different results on channel coordination required to perform numerical analysis in the subsequent section. Section 5 performs the numerical analysis to investigate the dynamics of coordination. Section 6 narrates a comparative discussion among the results obtained for different demand distributions. Section 7 summarises the findings, contribution and managerial implications of the study.

2. LITERATURE REVIEW

2.1 Buyback Contract

Pasternack (1985) introduces the concept of buyback contract which is proven to be a coordinating supply chain contract. He discusses the contract in context of a newsvendor problem. He proposes a model where unit credit is allotted to the newsvendor for each unsold item. Padmanabhan and Png (1995) argue the rationality to offer return policies to ensure that a supplier's image or brand value is not tarnished because of the discount policies adopted by the retailer for the leftover items. Supplier can make use of a buy back contract in order to manipulate competition among retailers. Emmons and Gilbert (1998) demonstrate buyback contracts with a price-setting newsvendor, while Taylor (2002) and Krishnan *et al.* (2004) make use of a combination of buyback contract and sales rebate contract and discuss the derived contract to coordinate the supply chain with respect to an effort dependent demand scenario. Xiao *et al.* (2009) analyse a coordination mechanism in a two stage supply chain with one manufacturer and one retailer facing consumer return. The study integrates consumer's return policy and manufacturer's buyback policy and designs a buyback or markdown money contract to coordinate the supply chain along with a partial refund policy.

Katok and Pavlov (2013) argue that the contracts used to coordinating a supply chain consisting one supplier and one retailer do not produce the desired results as predicted by theory. They find in a laboratory based experiment that the supplier's and retailer's decision making is affected by bounded rationality, aversion of inequality and incomplete information. Ozen *et al.* (2012) discuss channel coordination in a single manufacturer and multiple retailer scenario with respect to an extended buyback contract. In this case each of the retailers enjoys the scope of updating their respective orders after a certain lead time. Apart from the wholesale price and conventional buyback rate, one additional parameter is introduced at which the extra units are returned back by the manufacturer after the updation in the order quantity. The paper also discusses how the collaboration

among retailers by means of joint forecasting or forecast sharing impacts channel performance. Wu (2013) analyses the competition issues between two supply chains both consisting of one manufacturer and one retailer. The study examines the impact of buyback policy on wholesale price, order quantity and retail price of the supply chains. It shows that buyback policy drives higher profit compared to non buyback strategy in competing supply chains. Devangan *et al.* (2013) discuss the coordination issues in a single supplier, single retailer supply chain where the demand is dependent on the level of inventory displayed on retail shelf. The coordination discussed is based on a buyback contract and makes use of shapley value to consider fairness and individual rationality. He and Zhao (2012) discuss the production, replenishment and pricing decisions in three tier supply chain consisting of a raw material supplier, one manufacturer and one retailer. The study is conducted under the assumption of both demand and supply uncertainty and a returns policy (buyback contract) between the downstream players. It proposes how to design the terms of the contracts to obtain optimal supply chain profit. Zhao *et al.* (2014) discuss how level of uncertainty in market demand affects the applicability of buyback contract.

2.2 Quantity Flexibility Contract

Quantity flexibility contracts have come up as a combating mechanism to certain supply chain inefficiencies (Lee *et al.*, 1997) and its variety of uses have been discussed in the literature. The basic difference between a buyback contract and a quantity flexibility contract is that while in a buyback contract a retailer is partially protected for its entire leftover inventory whereas in case of a quantity flexibility contract he gets full credit for a portion of the leftover inventory. In a special case of quantity flexibility contract the retailer gets partial credit for a portion of the leftover inventory. This situation is sometimes referred to as backup agreement. Tsay (1999) discusses supply chain coordination for quantity flexibility contract. Backup agreement contracts are contracts studied by Pasternak (1985) and Eppen and Iyer (1997). The focus of the study done by Eppen and Iyer (1997) is not coordination but retailer's order quantity decision. Bassok and Anupindi (2008) provide an in-depth analysis of an important class of supply contracts called the Rolling Horizon Flexibility (RHF) contracts. They discuss and measure how much flexibility is sufficient and the effect of flexibility on customer satisfaction (captured by fill rate). Cachon and Lariviere (2001) and Lariviere (2002) study how forecast sharing between downstream retailer and upstream manufacturer and quantity flexibility contract interact with each other. They investigate the interplay of quantity flexibility contracts and information sharing. Lariviere (2002) discusses quantity flexibility contract where the supplier designs it in a manner to incentivize the retailer for better forecasting.

Cachon and Lariviere (2001) discuss, in connection with the quantity flexibility contract, that a downstream firm has a better demand forecast than the upstream supplier, but needs to convince the upstream supplier that her forecast is genuine. Downstream firm needs to exert the proper amount of effort to improve his demand forecast. Miltenburg and Pong (2007a) conduct similar study where the retailer selling a family of products has two order opportunities under

uncertain demand and demand forecast is updated between two orders using Bayesian method. Miltenburg and Pong (2007b) deals with the same problem with an additional assumption of capacity constraints for each order. Xiong *et al.* (2011) design a composite contract based on both buyback contract and quantity flexibility contract and prove that this contract is superior in terms of achieving channel coordination, profit allocation and risk allocation. Kim and Wu (2013) discuss a two-stage stochastic programming model for a quantity flexibility contract with respect to a two-echelon supply chain consisting of a supplier and a buyer. The buyer can place its orders both at the beginning of first period and second period. To adjust the order quantity at the beginning of second period, the buyer needs to buy an option contract.

Kim *et al.* (2014) discuss a model consisting of two or more suppliers and one buyer who informs the order size and a reservation quantity at the beginning of each period and can update the order quantity by adding or subtracting a portion of the reserved quantity after obtaining a forecast update. The study develops a linear programming model to analyze buyer's decision in the given context. Hening *et al.* (1997) analyze a quantity flexibility contract where the buyer enjoys a flexibility and he can order a volume which is lower than the committed volume at no additional cost where as he has to incur additional cost for ordering larger volume. Chung *et al.* (2014) introduce a contract by combining quantity flexibility contract and price-only discount contract and analyze its effectiveness in balancing the inventory risk.

2.3 Supply Chain Contract with Warranty and Associated Issues

Hu (2008) investigates the coordination mechanism of a single product in a single selling period with respect to a two stage supply chain. The analysis was done in case of a whole sale price contract and revenue sharing contract where a free replacement warranty is offered. The study investigates coordination mechanism through warranty period optimization and discusses design of other contract parameters to achieve coordination. The study is done in a risk-neutral domain. Ji *et al.* (2011) provides a coordination mechanism to coordinate the warranty decision and quality of the product in a supply chain consisting of one supplier and one manufacturer. The model built discusses the channel coordination issues along with a free replacement warranty. The study discusses three contracts: viz. First-best contract, Free replacement warranty with cost sharing and no cost sharing. Sinha and Sarmah (2011) analyse the dynamics of coordination and competition in a two-stage distribution channel where two different retailers compete on their retail price and warranty period offered to the customers to sell two substitutable products in the same market. The study shows the dynamics of change in retail price and warranty period with time and solves the problem by finding out the adjusted retail price and warranty period at equilibrium. The dynamics of competition is investigated where it is based on (i) exclusively on price, (ii) exclusively on warranty duration, (iii) both price and warranty duration. Dai *et al.* (2012) examines the issues related to interaction between warranty and quality related matters in a supply chain consisting of a supplier and a manufacturer. It investigates the solutions for channel coordination both in case of a centralized and a

decentralized supply chain. For the decentralized supply chain two separate cases are discussed viz. warranty decision taken by the supplier and the warranty decision taken by the manufacturer. Lan *et al.* (2014) analyze the buyer's decision problem related to pricing and warranty where the quality of the product is assumed to be a fuzzy variable.

3. MODEL

The manufacturer as the global coordinator of the supply chain needs to determine the necessary relationships among different contract parameters and warranty length in order to determine the optimal warranty length and other contract parameters. The manufacturer also needs to take into account about the nature of dependence of demand upon warranty length and the nature of demand. The manufacturer's problem is to determine the relationships among different contract parameters warranty length and other exogenous parameters. The supply chain taken in the current discussion is a two stage or two echelon supply chain consisting of a manufacturer and a retailer. There is perfect sharing of information. So, there exists information symmetry between the upstream and downstream player of the supply chain. The model involves selling a single product and the study is limited to single selling period. The product faces a stochastic demand i.e. demand uncertainty. Both the manufacturer and the retailer are rational. The supply chain faces demand uncertainty and the retailer needs to decide the order quantity before it faces the real demand. The manufacturer as the Stackelberg leader offers the terms of contract to the retailer by setting the buyback rate, wholesale price and warranty duration in case of buyback contract or fraction α , wholesale price and warranty duration in case of quantity flexibility contract. Based on Nandi (2014), the notations and their interpretation are given below. Risk is calculated by measuring the variance in the absolute profit.

w	Wholesale price per unit
c_m	Manufacturer's production cost per unit
c_r	Retailer's procuring cost per unit
c	Supply chain's total cost of production and distribution per unit
g_m	Manufacturer's cost of lost sales per unit
g_r	Retailer's cost of lost sales per unit
g	Total supply chain's cost of lost sales per unit.
v	Salvage value per unit
p	Retail price per unit
q	Number of units ordered
b	The buyback rate at which the manufacturer buys back the unsold units from the retailer
α	Parameter for quantity flexibility contract such that the manufacturer gives full credit to the retailer up to αq no of unsold units.
θ	Portion of the warranty cost borne by the retailer
k	Length of the warranty period
S(q,k)	Expected sales. In the current problem expected sales is a function the order quantity and warranty length since demand is a function of warranty length.
I(q,k)	Expected leftover inventory.

$I_r(q,k)$ Expected leftover inventory at the retailer’s end after implementation of return policy. This can be defined in case of quantity flexibility contract where a portion of the unsold inventory remains with the retailer also.

$L(q,k)$ Expected lost sales.
 $\mu(k)$ Expected total demand given the warranty length

r Cost of warranty per unit
 $f(X | K)$ Probability density function (Pdf) corresponding to the demand distribution for a given warranty length

$F(X | K)$ Cumulative distribution function (Cdf) corresponding to the demand distribution given a warranty length

$$\frac{p - w + g_r}{p - v + g - r \Pr(Y \leq k)} = \frac{(w - v)(1 - \alpha)F\{q(1 - \alpha)|k\} + c_r}{c - v}$$

and

$$1 - F(q^* | k) = \frac{c - v}{p - v + g - r \Pr(Y \leq k)}$$

4. RESULTS

4.1 Lemmas on Channel Coordination for Buyback Contract with Warranty

Lemma 1: The conditions for coordination of supply chain, in case of buyback contract with FRW policy, when the warranty cost is fully borne by the manufacturer are

$$\frac{p - b + g_r}{p - v + g - r \Pr(Y \leq k)} = \frac{c_r + w - b}{c - v}$$

$$1 - F(q^* | k) = \frac{c - v}{p - v + g - r \Pr(Y \leq k)}$$

4.2 Lemmas on Channel Coordination for Quantity Flexibility Contract with Warranty

Lemma 2: The conditions for coordination of supply chain, in case of quantity flexibility contract with FRW policy, when the warranty cost is fully borne by the manufacturer are

5. NUMERICAL ANALYSIS

Demand can be expressed as $x(k,d) = y(k) + d \dots$ (Eq. 1), where $y(k)$ is an increasing function in warranty length k and it captures the dependence of demand upon warranty

length. $y(k)$ can be expressed as $y(k) = \gamma - \delta k^{-\phi} \dots$ (Eq. 2), where γ, δ and ϕ are positive constants. This function is increasing and concave in k . Let f and F denote the probability density function (Pdf) and cumulative distribution function (Cdf) respectively of total demand represented by $x(k,d)$. Also, Let h and H denote the probability density function (Pdf) and cumulative distribution function (Cdf) respectively of d . Then F and H are related by $F(x | k) = H(x - y(k)) \dots$ (Eq. 3). It is also assumed that the time for the first failure of the product follows an exponential (β) distribution. The aforesaid demand function was given by Mills (1959) and used by several other researchers viz.: (Hu, 2008), (Dai *et al.*, 2012), who used this additive demand function in modelling warranty dependent demand for analysing coordination in case of quality and warranty related issues. The numerical analysis assumes that the entire warranty cost is borne by the manufacturer. It also assumes that the length of the warranty period may take only integer values if not mentioned otherwise. The objective function of the model is the total profit of the supply chain which is given below

$$\pi(q,k) = [p - v + g - r \Pr(Y \leq k)]S(q,k) - (c - v)q - g\mu(k) \dots \text{(Eq. 4)}$$

The constraint is $v < c_m < b < w < p \dots \dots \dots \text{(Eq. 5)}$

Table 1 List of exogenous parameters, their interpretations and numerical values

Parameters	Interpretations	Values
γ	Parameter included in the warranty dependent demand function	300
δ	Parameter included in the warranty dependent demand function	260
ϕ	Parameter included in the warranty dependent demand function	1.3
p	Retail price	100
c_m	Cost of manufacturing per unit	40
c_r	Cost of procurement per unit	10
c	Total supply chain’s cost of production and distribution per unit	50
g_m	Manufacturer’s cost of lost sales per unit	4
g_r	Retailer’s cost of lost sales per unit	7
g	Total supply chain’s cost of lost sales per unit	11
v	Salvage value per unit	20
r	Average cost of warranty per unit	50
β	Failure rate (hazard rate) of the product	0.05

5.1 Numerical Analysis Assuming Uniform Demand Distribution

Let us assume that $d \sim U(500,800)$ i.e. d follows a uniform distribution with lower limit 500 and upper limit 800.

5.1.1 Profit Maximization through Optimization of Warranty Period

Table 2 Values of Different Warranty Lengths, Corresponding Optimal Order Quantity and Supply Chain Profit

Warranty Length	Optimal Order Quantity	Supply Chain's Profit
1	738	29842
2	890	35268
3	931	35308
4	947	34282
5	955	32932
6	959	31481

Table 2 depicts the optimal order quantity and optimal supply chain profit corresponding to different warranty length. If the unit of warranty length is year, from the table it

is evident that the optimal warranty length is 3 years, the corresponding optimal order quantity is 931 units and the optimal supply chain profit is 35308. When the warranty length increases from 1 to 3 there is a gradual increase in the supply chain profit. On the other hand, when the warranty length increases from 3 to 6 there is a gradual decrease in the supply chain profit.

5.1.2 Dynamics of Coordination for Buyback Contract with Warranty

Table 3 represents different sets of values of buyback rate and wholesale price to achieve supply chain coordination. It also provides corresponding risk and profit shares of the manufacturer and the retailer. With increase in buyback rate, the retailer's profit decreases and the manufacturer's profit increases and the supply chain coordination is maintained. Also, with increase in the buyback rate the risk borne by the retailer decreases while the manufacturer bears more risk. It is also evident that with increase in buyback rate there is a linear increase in the wholesale price in order to achieve supply chain coordination. It is also evident that the relationship between the buyback rate and wholesale price is linear in nature. The profits for both the parties change linearly with increase in buyback rate while the risk for both the parties changes in a non linear fashion with increase in buyback rate. **Table 4** shows the profit earned in the case of a chosen suboptimal warranty period or suboptimal order quantity or both.

Table 3 Different sets of values of buyback rate and wholesale price to achieve coordination and corresponding risk-profit allocation for the manufacturer and the retailer

Buyback Rate	Wholesale Price	Retailer's Profit	Retailer's Profit (%)	Manufacturer's Profit	Manufacturer's Profit (%)	Retailer's Risk	Manufacturer's Risk	Total Supply Chain's Risk
43	55.85	28110	79.61	7198	20.39	1.3478×10^7	0.1067×10^7	2.2128×10^7
46	57.78	26500	75.05	8808	24.95	1.2097×10^7	0.1503×10^7	2.2128×10^7
49	59.71	24890	70.49	10418	29.51	1.0790×10^7	0.2014×10^7	2.2128×10^7
52	61.63	23290	65.96	12018	34.04	0.9558×10^7	0.2600×10^7	2.2128×10^7
55	63.56	21680	61.40	13628	38.60	0.8401×10^7	0.3261×10^7	2.2128×10^7
58	65.49	20070	56.84	15238	43.16	0.7318×10^7	0.3996×10^7	2.2128×10^7
61	67.42	18460	52.28	16848	47.72	0.6310×10^7	0.4806×10^7	2.2128×10^7
64	69.35	16850	47.72	18458	52.28	0.5376×10^7	0.5690×10^7	2.2128×10^7
67	71.28	15240	43.16	20068	56.84	0.4518×10^7	0.6649×10^7	2.2128×10^7
70	73.21	13630	38.60	21678	61.40	0.3734×10^7	0.7683×10^7	2.2128×10^7
73	75.14	12020	34.04	23288	65.96	0.3024×10^7	0.8792×10^7	2.2128×10^7
76	77.07	10410	29.48	24898	70.52	0.2390×10^7	0.9975×10^7	2.2128×10^7

Table 4 Values of Different Sub-Optimal Warranty Lengths, Sub-Optimal Order Quantities and Corresponding Supply Chain Profit

Warranty Length	Order Quantity	Supply Chain's Profit
1	900	25986
1	960	22592
2	880	35253
2	980	34105
4	870	33466
4	1000	33902
5	860	31722
5	1050	31738
6	850	29925
6	1075	29742

One of the important conditions of achieving coordination in a supply chain by using supply chain contract is the contract should be capable of allocating profit in an arbitrary manner between the players of the supply chain. In the current problem where the coordination mechanism is discussed in case of buyback contract and quantity flexibility contract along with a free replacement warranty, the issue of arbitrary profit allocation is discussed. The contract parameters and warranty length can be manipulated to achieve any predetermined profit allocation between the members of the supply chain. At the same time, optimal supply chain profit can be achieved. The portion of profit enjoyed by the manufacturer and the retailer is a function of their bargaining power and also of the nature of the product concerned and the market they are operating in. The choice of contract undertaken by the parties of the supply chain in the above two

cases depends on the business objectives of the manufacturer or supplier and the nature of supply chain it is operating.

5.1.3 Sensitivity Analysis

In this section, sensitivity analysis is performed to find out the impact of change in different parameters (mean, standard deviation) of demand distribution on the optimal order quantity, optimal warranty length, supply chain profit and supply chain risk. It also investigates the effect of change in different exogenous parameters such as cost of production

and average warranty cost, retail price on optimal order quantity, optimal warranty length, supply chain profit and supply chain risk. Lastly, the impact of change in different parameters of the warranty dependent demand functions upon optimal order quantity, optimal warranty length and supply chain profit is examined.

5.1.3.1 Sensitivity Analysis with Respect to Parameters of the Demand Distribution

Table 5 Effect of change in standard deviation on optimal order quantity, optimal warranty length, optimal supply chain profit (Warranty lengths can assume values from 1 to 6 with a step size 0.2)

Lower Limit	Upper Limit	Standard Deviation (σ)	Optimal Order Quantity	Optimal Warranty Length	Supply Chain's Profit
550	750	57.74	896	2.4	36489
525	775	72.17	904	2.4	36002
500	800	86.60	911	2.4	35516
475	825	101.04	919	2.4	35030
450	850	115.47	926	2.4	34543
400	900	144.34	941	2.4	33571
350	950	173.21	956	2.4	32598
300	1000	202.07	971	2.4	31625
250	1050	230.94	992	2.6	30653
200	1100	259.81	1007	2.6	29683

Table 5 examines the impact of change in standard deviation in demand upon the optimal order quantity, optimal warranty length and supply chain profit when mean of demand remains unchanged. In this analysis the warranty period is allowed to assume fractional values (values from 1 to 5 with a step size of 0.2). It is observed that with increase

in the standard deviation there is a increase in the optimal warranty length. For example, when the standard deviation of demand is increased from 57.74 to 230.94, there is an increase in the optimal warranty length from 2.4 to 2.6. Therefore, additional variance introduce into the system will result in an increase in the optimal warranty length.

Table 6 Effect of change in mean on optimal order quantity, optimal warranty length, optimal supply chain profit and supply chain risk

Lower Limit	Upper Limit	Mean (μ)	Coefficient of Variation (σ/μ)	Optimal Order Quantity	Optimal Warranty Length	Supply Chain's Profit	Supply Chain's Risk
550	850	700	0.12	940	2	37530	2.4053×10^7
525	825	675	0.13	915	2	36399	2.4053×10^7
475	775	625	0.14	906	3	34232	2.2128×10^7
450	750	600	0.14	881	3	33156	2.2128×10^7
425	725	575	0.15	856	3	32080	2.2128×10^7
400	700	550	0.16	831	3	31004	2.2128×10^7

Table 6 represents the impact of change in the mean of the demand distribution upon the optimal order quantity, optimal warranty length, supply chain profit and supply chain risk when the standard deviation of the distribution is kept unchanged. With increase in the mean of the demand, there is an increase in the optimal order quantity and supply chain profit. It is also observed that with increase in mean, the optimal warranty length either decreases or it remains the same. With increase in the mean of the demand there is an increase in the supply chain risk only when there is a decrease

in the optimal warranty length; otherwise the supply chain risk remains constant within the same optimal warranty length. It is also evident that with increase in mean, the supply chain profit increases linearly. The results provide insight for managerial practice by showing that whenever there is a positive impetus in demand due to increase in mean of the stochastic part of the demand distribution the coordinator either needs to decrease to warranty length or keep it unchanged depending on the increase in the mean of demand in order to enhance the channel performance.

5.1.3.2 Sensitivity Analysis with Respect to Exogenous Parameters

It is evident from **Table 7** that with increase in cost of production and average warranty cost per unit (the changes in

these two exogenous parameters have been considered together, since these two parameters change simultaneously) the supply chain profit decreases and there is a decrease in the optimal warranty length.

Table 7 Impact of change in cost of production and average warranty cost on optimal order quantity, optimal warranty length, supply chain profit and supply chain risk

Cost of production & average warranty cost	Optimal Order Quantity	Optimal Warranty Length	Supply Chain's Profit	Supply Chain's Risk
35	985	3	51513	3.6212×10 ⁷
40	967	3	46017	3.1675×10 ⁷
45	949	3	40614	2.6891×10 ⁷
50	931	3	35308	2.2128×10 ⁷
55	872	2	30471	1.9343×10 ⁷
60	854	2	25769	1.4981×10 ⁷
65	835	2	21162	1.0911×10 ⁷

Table 7 also shows that with increase in the cost of production and average warranty cost per unit, there is a decrease in the supply chain risk as well as supply chain profit. Also, the optimal warranty length decreases to 2. It

also shows that there is a sudden drop in the optimal order quantity when the cost of production and average warranty cost per unit increase from 50 to 55. This corresponds to the drop in the optimal warranty length from 3 to 2.

Table 8 Impact of change in retail price on optimal order quantity, optimal warranty length, supply chain profit and supply chain risk

Retail Price	Optimal Order Quantity	Optimal Warranty Length	Supply Chain's Profit	Supply Chain's Risk
85	868	2	22931	1.1939×10 ⁷
90	876	2	27029	1.5464×10 ⁷
95	884	2	31142	1.9652×10 ⁷
100	931	3	35308	2.2128×10 ⁷
105	937	3	39656	2.6884×10 ⁷
110	942	3	44013	3.1971×10 ⁷
115	947	3	48379	3.7653×10 ⁷

Table 8 depicts the impact of change in retail price on the optimal order quantity, optimal warranty length, supply chain profit and supply chain risk. From **Table 8** it is evident that increase in retail price results in increase in the optimal warranty length and optimal order quantity. It also increases the supply chain profit and risk borne by the supply chain. Within the same optimal warranty length, the optimal order quantity increases linearly with increase in the retail price. The optimal supply chain profit also increases linearly with increase in the retail price.

5.1.4 Dynamics of Coordination for Quantity Flexibility Contract

Table 9 represents different pairs of values of fraction α and wholesale price w in order to achieve channel coordination. It is observed from this table that with increase in α , there is an increase in the wholesale price in order to achieve coordination. Also, with increase in α there is a decrease in the retailer's profit, risk and increase in the manufacturer's profit, risk while the supply chain coordination is maintained.

Table 9 Different sets of values of Fraction α (<1) and wholesale price to achieve coordination and corresponding risk-profit allocation for the manufacturer and the retailer

Fraction α (<1)	Wholesale Price	Retailer's Profit	Retailer's Profit (%)	Manufacturer's Profit	Manufacturer's Profit (%)	Retailer's Risk	Manufacturer's Risk	Total Supply Chain's Risk
0.08	49.19	34009	96.32	1299	3.68	1.8211×10^7	0.0374×10^7	2.2128×10^7
0.10	52.02	31702	89.79	3606	10.21	1.5433×10^7	0.0841×10^7	2.2128×10^7
0.12	55.31	28988	82.10	6320	17.90	1.2522×10^7	0.1625×10^7	2.2128×10^7
0.14	59.17	25767	72.98	9541	27.02	0.9603×10^7	0.2826×10^7	2.2128×10^7
0.16	63.76	21895	62.01	13413	37.99	0.6824×10^7	0.4553×10^7	2.2128×10^7
0.18	69.27	17198	48.71	18110	51.29	0.4371×10^7	0.6904×10^7	2.2128×10^7
0.20	76.00	11401	32.29	23907	67.71	0.2420×10^7	0.9916×10^7	2.2128×10^7
0.22	84.37	4121	11.67	31187	88.33	0.1066×10^7	1.3444×10^7	2.2128×10^7

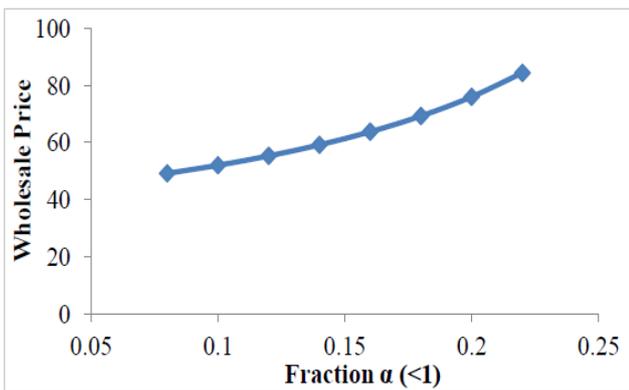


Figure 1 Relation between Fraction α (<1) and Wholesale Price for Achieving Coordination

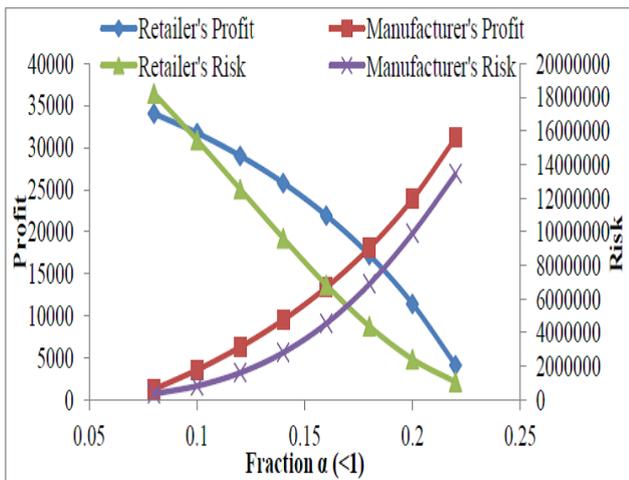


Figure 2 Risk and Profit Allocation for the Manufacture and Retailer for Different Values of α

Figure 1 represents the nature of relation between the fraction α and wholesale price in order to achieve channel coordination. Wholesale price versus α graph is convex increasing in nature. **Figure 2** shows that with increase in α , retailer's profit is concave decreasing and retailer's risk is convex decreasing in nature. It also shows that with increase in α manufacturer's profit as well as risk is convex increasing in nature.

5.2 Numerical Analysis Assuming Normal Demand Distribution

Let us assume that $d \sim N(\mu, \sigma^2)$ where $\mu=650$ and $\sigma=100$ i.e. d follows a normal distribution with mean 650 and standard distribution 100.

5.2.1 Dynamics of Coordination for Quantity Flexibility Contract with Warranty

From **Table 10** different sets of values of fraction α and wholesale price are obtained in order to achieve channel coordination. It is observed from this table that with increase in α , there is an increase in the wholesale price in order to achieve coordination. But the wholesale price is constant once α attains a threshold value. Beyond this value even if there is an increase in α , the wholesale price remains constant. Also, with increase in α there is a decrease in the retailer's profit, risk and increase in the manufacturer's profit, risk while the supply chain coordination is maintained. Again, beyond the aforesaid threshold value of α , increase in α does not have any impact on risk and profit of both the parties in the supply chain. This phenomenon is observed only in case of normal distribution among the distributions considered in this study.

Table 10 Different sets of values of Fraction α (<1) and wholesale price to achieve coordination and corresponding risk-profit allocation for the manufacturer and the retailer

Fraction α (<1)	Wholesale Price	Retailer's Profit	Retailer's Profit (%)	Manufacturer's Profit	Manufacturer's Profit (%)	Retailer's Risk	Manufacturer's Risk	Total Supply Chain's Risk
0.10	54.20	29516	84.17	5550	15.83	1.9677×10^7	0.0870×10^7	2.6131×10^7
0.15	63.22	21998	62.73	13068	37.27	1.2061×10^7	0.3437×10^7	2.6131×10^7
0.20	71.11	15376	43.85	19690	56.15	0.6476×10^7	0.7345×10^7	2.6131×10^7
0.25	75.96	11289	32.19	23777	67.81	0.3664×10^7	1.0666×10^7	2.6131×10^7
0.30	78.08	9499	27.09	25567	72.91	0.2600×10^7	1.2408×10^7	2.6131×10^7
0.35	78.77	8915	25.42	26151	74.58	0.2272×10^7	1.3040×10^7	2.6131×10^7
0.40	78.95	8762	24.99	26304	75.01	0.2185×10^7	1.3215×10^7	2.6131×10^7
0.45	78.98	8737	24.92	26329	75.08	0.2167×10^7	1.3250×10^7	2.6131×10^7
0.50	78.99	8728	24.89	26338	75.11	0.2163×10^7	1.3259×10^7	2.6131×10^7
0.55	78.99	8728	24.89	26338	75.11	0.2163×10^7	1.3259×10^7	2.6131×10^7
0.60	78.99	8728	24.89	26338	75.11	0.2163×10^7	1.3259×10^7	2.6131×10^7

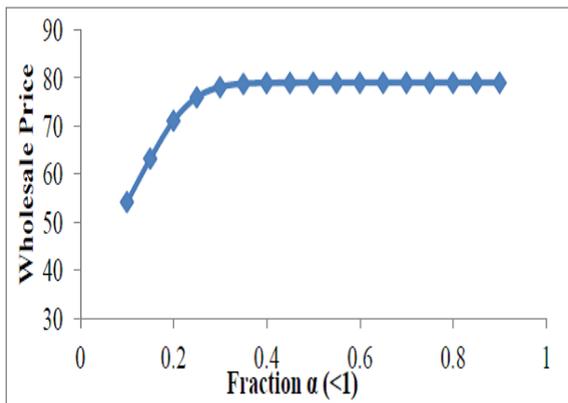


Figure 3 Relation between Fraction α (<1) and Wholesale Price for Achieving Coordination

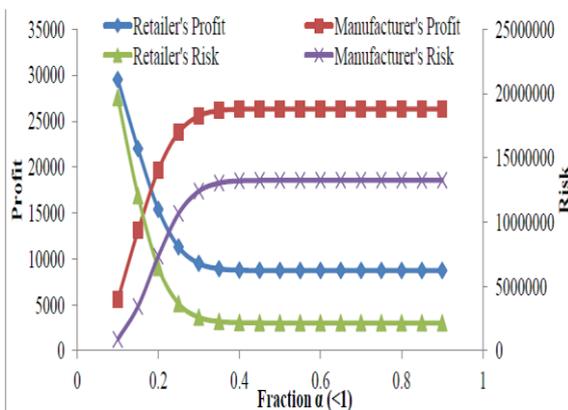


Figure 4 Risk and Profit Allocation for the Manufacture and Retailer for Different Values of α

Figure 3 represents the nature of relation between the fraction α and wholesale price in order to achieve channel coordination. The curve representing the wholesale price versus α graph is concave increasing in nature. For $\alpha \geq 0.50$, Fig. 3 shows that the wholesale price remains constant with increase in α . **Figure 4** shows the relationships of risk and profit for both the parties with α . With increase in α , retailer's profit and risk are convex decreasing in nature. It also shows that with increase in α manufacturer's profit is concave increasing in nature and manufacturer's risk is nonlinear increasing in nature.

5.3 Numerical Analysis Assuming Exponential Demand Distribution

Let us assume that $d \sim \text{Exp}(\lambda)$ i.e. d follows an exponential distribution with $\lambda=1/650$. i.e. the mean of the distribution is $1/\lambda=650$.

5.3.1 Dynamics of Coordination for Quantity Flexibility Contract with Warranty

Table 11 represents different pairs of α and wholesale price for achieving channel coordination. It also shows corresponding risk and profit shares for the retailer and the manufacturer. The table also shows that with increase in α , the wholesale price also increases to maintain supply chain coordination. With increase in α , there is a decrease in retailer's profit as well as risk and there is an increase in the manufacturer's profit as well as risk. In case of exponential demand distribution, the manufacturer is required to offer a higher flexibility to the retailer in terms of quantity ordered by fixing a relatively larger α to ensure that both the parties in the supply chain have a positive profit. This is an important managerial insight for the manufacturer for deciding the amount of flexibility to offer when the retailer faces an exponential demand distribution.

Table 11 Different sets of values of Fraction α (<1) and wholesale price to achieve coordination and corresponding risk-profit allocation for the manufacturer and the retailer

Fraction α (<1)	Wholesale Price	Retailer's Profit	Retailer's Profit (%)	Manufacturer's Profit	Manufacturer's Profit (%)	Retailer's Risk	Manufacturer's Risk	Total Supply Chain's Risk
0.35	53.49	17111	94.46	1004	5.54	2.1765×10^8	0.1128×10^8	3.0885×10^8
0.40	56.15	15805	87.25	2310	12.75	1.8724×10^8	0.1908×10^8	3.0885×10^8
0.45	59.07	14336	79.14	3779	20.86	1.5602×10^8	0.3017×10^8	3.0885×10^8
0.50	62.26	12686	70.03	5429	29.97	1.2501×10^8	0.4519×10^8	3.0885×10^8
0.55	65.70	10853	59.91	7262	40.09	0.9560×10^8	0.6456×10^8	3.0885×10^8
0.60	69.31	8867	48.95	9248	51.05	0.6947×10^8	0.8801×10^8	3.0885×10^8
0.65	72.99	6763	37.33	11352	62.67	0.4816×10^8	1.1431×10^8	3.0885×10^8
0.70	76.54	4634	25.58	13481	74.42	0.3290×10^8	1.4032×10^8	3.0885×10^8
0.75	79.70	2607	14.39	15508	85.61	0.2395×10^8	1.6077×10^8	3.0885×10^8
0.80	82.14	858	4.74	17257	95.26	0.2052×10^8	1.6837×10^8	3.0885×10^8

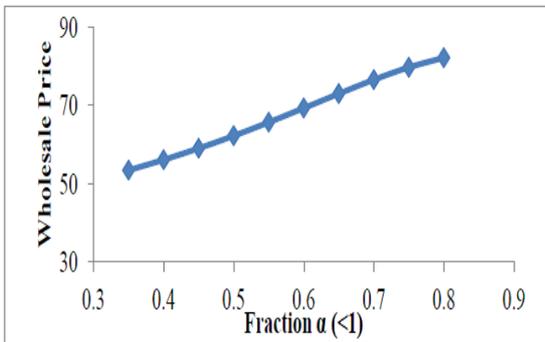


Figure 5 Relation between Fraction α (<1) and Wholesale Price for Achieving Coordination

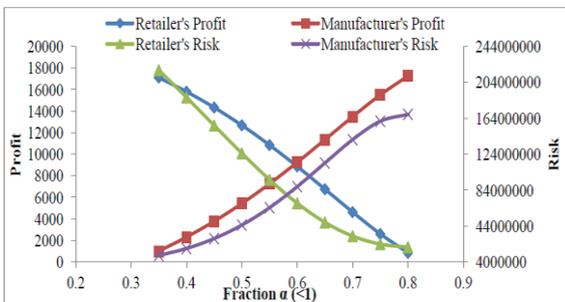


Figure 6 Risk and Profit Allocation for the Manufacturer and Retailer for Different Values of α

Figure 5 shows that with increase in α , the wholesale price increases in a nonlinear fashion. The nature of the wholesale price versus α graph is nonlinear increasing. **Figure 6** graphically represents the nature of change of risk as well as profit of both the retailer and the manufacturer with increase in α .

5.3.2 Sensitivity Analysis with Respect to Exogenous Parameters

Table 12 shows that with increase in the cost of production and average warranty cost the decrease in optimal order quantity and supply chain profit happens at a much faster rate and nonlinear fashion. The impact of change in retail price is more significant in case of exponential distribution compared to uniform and normal distribution as evident from **Table 13**.

Table 12 Impact of Change in Cost of Production and Average Warranty Cost on Optimal Order Quantity, Optimal Warranty Length, Supply Chain Profit and Supply Chain Risk

Cost of production & average warranty cost	Optimal Order Quantity	Optimal Warranty Length	Supply Chain's Profit	Supply Chain's Risk
35	1382	4	36335	8.1452×10^8
40	1181	3	29439	6.2054×10^8
45	1031	3	23423	4.4126×10^8
50	907	3	18115	3.0885×10^8
55	802	3	13407	2.1019×10^8
60	687	2	9268	1.6317×10^8
65	606	2	5792	1.0552×10^8

Table 13 Impact of Change in Retail Price on Optimal Order Quantity, Optimal Warranty Length, Supply Chain Profit and Supply Chain Risk

Retail Price	Optimal Order Quantity	Optimal Warranty Length	Supply Chain's Profit	Supply Chain's Risk
85	757	2	8671	1.4478×10^8
90	825	3	11709	1.7421×10^8
95	867	3	14873	2.3546×10^8
100	907	3	18115	3.0885×10^8
105	945	3	21427	3.9491×10^8
110	980	3	24800	4.9270×10^8
115	1014	3	28228	6.0484×10^8

6. COMPARISON AMONG THE RESULTS OBTAINED FOR UNIFORM, NORMAL AND EXPONENTIAL DEMAND DISTRIBUTION

Observing the results obtained by performing numerical and sensitivity analysis separately for uniform, normal and exponential distribution, some differences are noted. The risk of the total supply chain when channel coordination is achieved is much higher in case of exponential distribution

compared to those for the uniform and normal distribution. In case of the exponential distribution the order of magnitude for the total supply chain risk is 8 whereas the same is 7 in case of uniform and normal distributions. This can also be explained by noting the differences in standard distribution for these distributions.

It is also interesting to note that with increase in the cost of production and average warranty cost the decrease in optimal order quantity and supply chain profit happens at a much faster rate and nonlinear fashion for exponential distribution compared to in case of uniform and normal distribution where the decrease happens at a slower rate and linearly. Therefore, for different parties of the supply chain and for the supply chain as a whole, change in the cost of production and average warranty cost has a larger implication compared to other distributions when the demand distribution is exponential. The same is the case, in context of observing the impact of change in retail price on the optimal order quantity and supply chain profit. The impact is more significant in case of exponential distribution compared to uniform and normal distribution. This may also be attributed to the reason that though the means for all the three distributions have been kept similar to maintain parity, the standard deviation (SD) of the exponential distribution is much higher compared to the SDs of the other two distributions. This is because, in case of exponential distribution the mean and SD of the distribution are always equal irrespective of the chosen parameter of the distribution.

In case of designing contract parameters for quantity flexibility contract, a difference among the three distributions is observed in terms of the available range of α for which both the parties in the supply chain enjoy a positive profit. The available range of α is largest for normal distribution and smallest for uniform distribution. Again, the available range of α in case of exponential distribution is greater than that of uniform distribution and smaller than that of normal distribution. Beyond the aforesaid available range of α , the profit of either of the party in the supply chain becomes negative. At the time of designing the contract parameters, this available range of α is to be considered in order to frame a realistic contract where both the parties at least earn a positive profit. These observations conform to the findings that the coordination in case of quantity flexibility contract differs depending on the demand distribution faced by the retailer. It is also observed in case of exponential demand distribution with higher variance that the manufacturer is required to offer a higher flexibility to the retailer in terms of quantity ordered by fixing a relatively larger α to ensure that both the parties in the supply chain have a positive profit. This is an important managerial insight for the manufacturer for deciding the amount of flexibility to offer when the retailer faces an exponential demand distribution.

The reason behind the longest range of availability for parameter α in case of normal distribution is the existence of a threshold value of α beyond which further increase of its value does not have any impact on the profit or risk for both the parties in the supply chain. This phenomenon is not observed in case of uniform or exponential distribution. Again, in case of normal distribution there exists a threshold value of α beyond which further increase in the value of α does not impact the value of the wholesale price to be set in

order to achieve channel coordination. This is not observed in case of uniform or exponential distribution.

7. CONCLUSION

In the current problem where the coordination mechanism is discussed in case of buyback contract and quantity flexibility contract along with a free replacement warranty, the issue of arbitrary profit allocation is discussed. The contract parameters and warranty length can be manipulated to achieve any predetermined profit allocation between the members of the supply chain. At the same time, optimal supply chain profit can be achieved. The portion of profit enjoyed by the manufacturer and the retailer is a function of their bargaining power and also of the nature of the product concerned and the market they are operating in.

The study has the following managerial implications/contributions

- The study contributes to the existing literature by deriving necessary conditions for achieving supply chain coordination in case of a buyback contract and a quantity flexibility contract in conjunction with warranty. The conditions involve relationships among the contract parameters and different decision variables.
- The study provides necessary guidelines to the supply chain coordinator to achieve optimal supply chain profit by appropriate design of the contract parameters and warranty period optimization among a given set of possible warranty lengths. The study also investigates the risk-profit allocation of the parties in the supply chain with change in the contract parameter (buyback rate in case of buyback contract, α in case of quantity flexibility contract).
- Wide-spread sensitivity analysis explores the impact of change in the parameters of demand distributions (mean, standard deviation, coefficient of variation), exogenous variables (cost of production, average warranty cost, retail price) and parameters of the warranty dependent demand function upon the optimal order quantity, optimal warranty length, supply chain profit and supply chain risk.
- The study provides an insight to the managerial practice by establishing that when there is a positive impact in the demand due to increase in the mean of the stochastic part of the demand distribution, the supply chain earns more profit by reducing the warranty length or by keeping it unchanged.
- The study contributes by providing another insight to managerial practice by showing that in case of exponential demand distribution with higher variance, the manufacturer is required to offer a higher flexibility to the retailer in terms of quantity ordered by fixing a relatively larger flexibility parameter to ensure that both the parties in the supply chain have a positive profit.

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Shirsendu Nandi is an assistant professor in the area of operations in Indian Institute of Management Rohtak. Shirsendu Nandi is FPM (Fellow in Management) from Indian Institute of Management Indore. He also holds Master of Science in mathematics from IIT Kharagpur. He has done his Bachelor of Science with mathematics honours from Presidency college under University of Calcutta. He is the recipient of prestigious N.B.H.M (National Board for Higher Mathematics) Scholarship awarded by Department of Atomic Energy, Govt. of India. He has worked for two years with Wipro Technologies in the area of business intelligence. His teaching interests include subjects like Quantitative Methods, Business Statistics, Advanced Probability and Stochastic Process, Supply Chain Management, Operations Management, Advanced Optimization, Managerial Economics etc. His research interests include Supply chain contracts, Supply chain coordination, Supply chain modelling and network design, Mathematical model building for supply chain and operations related problems.

APPENDIX:**Proof of Lemma 4.1:**

$$\begin{aligned}\pi_r &= pS(q, k) + bI(q, k) - wq - c_r q - g_r L(q, k) \\ &= (p - b + g_r)S(q, k) + (b - w - c_r)q - g_r \mu(k) \\ \therefore \frac{\partial \pi_r}{\partial q} &= (p - b + g_r) \frac{\partial S}{\partial q} + (b - w - c_r)\end{aligned}$$

$$\therefore \frac{\partial^2 \pi_r}{\partial q^2} = (p - b + g_r) \frac{\partial^2 S}{\partial q^2} = (p - b + g_r)[-f(q|k)] < 0$$

Therefore, π_r is concave in q

Now,

$$\frac{\partial \pi_r}{\partial q} = 0 \Rightarrow \frac{\partial S}{\partial q} = \frac{c_r + w - b}{p - b + g_r}$$

Again,

$$\pi(q, k) = [p - v + g - r \Pr(Y \leq k)]S(q, k) - (c - v)q - g \mu(k)$$

$$\frac{\partial^2 \pi}{\partial q^2} = [p - v + g - r \Pr(Y \leq k)] \frac{\partial^2 S}{\partial q^2} = [p - v + g - r \Pr(Y \leq k)][-f(q|k)] < 0$$

$$\therefore \frac{\partial \pi}{\partial q} = 0 \Rightarrow \frac{\partial S}{\partial q} = \frac{c - v}{p - v + g - r \Pr(Y \leq k)}$$

Therefore, the required conditions are

$$\begin{aligned}\frac{p - b + g_r}{p - v + g - r \Pr(Y \leq k)} &= \frac{c_r + w - b}{c - v} \\ 1 - F(q^*|k) &= \frac{c - v}{p - v + g - r \Pr(Y \leq k)}\end{aligned}$$

Proof of Lemma 4.2

$$\begin{aligned}\pi_r &= pS(q, k) + wE[\min((q - x)^+, \alpha q)] - wq - c_r q - g_r L(q, k) + vI_r(q, k) \\ &= (p - w + g_r)S(q, k) - \{w(1 - \alpha) - v(1 - \alpha) + c_r\}q - g_r \mu(k) + (w - v)S\{q(1 - \alpha), k\}\end{aligned}$$

Since,

$$E[\min((q - x)^+, \alpha q)] = q - S(q, k) + S\{q(1 - \alpha), k\} - q(1 - \alpha)$$

$$\frac{\partial \pi_r}{\partial q} = (p - w + g_r) \frac{\partial S}{\partial q} - \{(w - v)(1 - \alpha) + c_r\} + (w - v) \frac{\partial S^*}{\partial q}$$

$$\text{where, } S^* = S\{q(1 - \alpha), k\}$$

$$\begin{aligned} \therefore \frac{\partial^2 \pi_r}{\partial q^2} &= (p - w + g_r) \frac{\partial^2 S}{\partial q^2} + (w - v) \frac{\partial^2 S^*}{\partial q^2} \\ &= (p - w + g_r)[-f(q|k)] - (w - v)(1 - \alpha)^2 f\{q(1 - \alpha)|k\} < 0 \end{aligned}$$

Therefore, π_r is concave in q

Now,

$$\frac{\partial \pi_r}{\partial q} = 0 \Rightarrow \frac{\partial S}{\partial q} = \frac{(w - v)(1 - \alpha)F\{q(1 - \alpha)|k\} + c_r}{p - w + g_r}$$

Also,

$$\frac{\partial \pi}{\partial q} = 0 \Rightarrow \frac{\partial S}{\partial q} = \frac{c - v}{p - v + g - r \Pr(Y \leq k)}$$

$$\frac{\partial S}{\partial q} = 1 - F(q|k)$$

Hence, the proof