OPERATIONS AND SUPPLY CHAIN MANAGEMENT
Vol. 11, No. 4, 2018, pp. 226 - 236
ISSN 1979-3561 | EISSN 2759-9363

An Optimal Model at the Interface of Operations with Social Media Marketing

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ABSTRACT

Social media sales and the logistics of products is being shuttled into a new technological paradigm. The purpose of this paper is to review the role of social media sales force and its effects on lot size process and the pricing under an infinite-time horizon assumption. We begin with reviewing the role and importance of social media in business; with a focus on the mathematic deterministic model for decision making that offer ways in which social media influences each step. Scenarios are specifically considered. In the first scenario, price is determined by the market. What size to hold inventoried products and what to budget for promotions on a social media platform are the important variables in market determined pricing. In the second scenario, price is examined by the decision maker based on maximizing the profit margin. In this first case scenario, advertising and product prices are jointly considered to maximize the profit margin. In this second case scenario, lot sizing is considered to minimize costs of holding inventories and placing orders. In both scenarios, the profit function outcome is concave. Therefore, the model optimizes solutions for pricing, lot-sizing for social media advertising policies.

Keywords: *applied mathematics, social media advertisement, pricing, lot sizing, deterministic optimization model, operations and marketing interface*

1. INTRODUCTION AND LITERATURE REVIEW

With the power of social media applications at the consumer level, the use of media is now viewed as a primary sales channel. Marketers have embraced the ability of social media tools such as Facebook to spread their message. Now with a fully integrated sales strategy and real-time pricing the decision for many businesses to use social media to advertise is obvious. However, with any decision to use today's technology accompanied with the new sales tools, there comes relevant unanswered questions. Whether or not the business becomes a marketing success using social media and exceeds sales expectations is dependent on other conditions. These new problematic questions will present themselves. What are the outcomes if:

- prices are decreased on the media platform?
- advertising is increased on the media platform?
- nothing is done with the lot size?

"What ifs" come with risk, and the question is, do these risks out-way the reward? The Retail and Hospitality industries rely heavily on social media in today's market. The ability to know and optimize advertising budgets reflects order quantity. This is not only exceptionally valuable to keep risks minimized, it is a tool of survival, especially for start-ups that are looking to use social media platforms to launch new products. Moreover, advertising can be increased or decreased easily with the click of a button making this model extremely valuable in minimizing possible negative effects on lot sizes and logistics. Social media (SM) in the twentieth century plays a large role in advertising, demand for products, and lot sizes.

As discussed by Weinberg and Pehlivan E (2011), traditional organization determines a set budget amount. Marketers then decide how much to spend on various media channels that directly deliver the messages to customers. Given the relationship-oriented nature of social media, the spending process is characterized as investment in establishing, building or maintaining relationships, generating positive word of mouth, engaging conversations about products etc. In other words, social media marketers focus is on spending the same budget as traditional marketing in a way that will result in others carrying or distributing their 'own' messages that are related to the marketer's brand. The fact is social media advertising investment enhances customer engagement behavior positively and thus increases sales. As evidenced in the movie file industries, Chong et al (2017) used ordinary least square regression models and reported that enhanced customer engagement on Facebook and YouTube positively correlates with movie revenue. The fact is consumers are spending more time on cell-phone, i-Pad, computer, and other technology with many turning away from traditional media such as television, radio, magazines, and newspapers, they are turning increasingly more toward social media engagement. Consumers use social media to search for product information, promotions and deals, and to make purchasing decisions. Witnessing the power of social media and consumer behaviors, marketers use social media to advertise new products and services, offer discounts, and enhance product awareness and sales. Andzulis et al (2012) provided examples of Ford Motor Company employing social media to promote the release of the new model Ford Focus. Chief executive officer (CEO) Robert McDonald reports his ad budget due Facebook and Google potentially is more efficient than the traditional media.

Our work not only incorporates the power of social media advertisements on sales, but also considers marketing and operations interface decisions. Operations and marketing have long known to be two crucial functional areas that contribute to business success. Marketing is the creation and/or maintenance of customer demand. Operations is the supply and fulfillment of customer demand at minimal costs with above-average quality products. With no surprise marketing and operations are tightly connected in many businesses. So, to respond to customer demands quickly with high satisfaction and enhanced business profits, both operations and marketing must be considered. If those internal functions focus on their own goals in an attempt to maximize their own success (known as "local optimization") this may hurt the other function's performance and eventually damage the success of the entire business. Specifically, when they are in conflict, one often sees a mismatch in demand and supply, this leads to production inefficiencies, unsatisfied customers, and deteriorating corporate profit. Therefore, the active collaboration and communication between marketing and production have emerged as an important research area.

This work contributes by proposing the mathematic deterministic model which allows the operation and marketing components to be considered jointly and optimally, rather than individually and separately; optimal performance in these two areas using this research model is maximized, thus increasing annual profit. Our work differs from previous research work in that our model allows the decision makers to know how much is needed and when to produce/order while incorporating the influence of social media advertising on the demand function. In addition, the budget needed for SM advertising and pricing is addressed to optimize business success without over spending. Note that the advertising with SM non-price promotions, for example, coupons from SM channels for advertising and other SM non-price sale efforts are included in the SM advertising cost.

The importance of coordinating marketing and operations functions has motivated a number of research scholars resulting in accumulated research work in the past and recent years. The research is classified into three research streams: (1) joint pricing and inventory control problems, (2) promotion and inventory control problems, and (3) a combination of the first two streams, that is, the pricing, promotion and inventory control problems.

The research of joint pricing and inventory control was originated by the classical work of Whitin (1955) which expressed their price and demand relationships in additive and multiplicative models. Dada and Petruzzi (1999) extended the price-demand relationship in both multiplicative and additive models to the newsvendor context. They proposed a new benchmark to structure the optimal policy. Whitin's (1955) demand functions were later used by Karlin and Carr (1962), and Mills (1962). Federgruen and Heching(1999) addressed the stochastic-demand multi-period models by assuming that the price can be adjusted dynamically. They showed that the optimal policy is a base-stock-list-price policy. Karakul (2008) and Webster and Weng (2008) also considered similar problems but specific to the fashion industry. GürayGüler et al (2014) applied to a service facility with capacity constrained (such as the facility for automobile after-sales services) and with several customer classes. Aydin and Porteus (2008) considered the model similar to Dada and Petruzzi's (1999) with an additional scope to include multiple products in a given assortment, under price-based substitution. Lu et al (2016) developed the K-approximate convexity, to solve this inventory-pricing coordination problems and to provide well-structured heuristic policies.

The second stream, the joint promotion and inventory control problem was studied by Balcer (1980, 1981); they established the properties of inventory and advertising optimal policy to minimize the expected discounted cost over a finite horizon by assuming the demand being influenced by a level of goodwill. Their later paper suggested the optimal solution for the discrete-time dynamic-inventory and advertising models by assuming goodwill increases with advertising expense exponentially over time and demands distribution depending on goodwill. Sogomonian and Tang (1993) proposed the optimization for promotion and dynamic inventory decision by using the mixed integer programming and applying a longest path algorithm in which the demand is influenced by the last promotion effort and retail pricing. They compared the profit made from the coordinated decision and that of non-coordination; as expected, the coordinated profit outperforms the non-coordination. No competitive response is discussed in their model. Sethi and Zhang (1994) and Cheng and Sethi (1999) developed the stochastic production-promotion model and assumed the stochastic demand is affected by promotion. They used the Markov decision process (MDP) to model the joint decision of promotion and inventory. Their demand levels and the corresponding distribution of random demand are promotion-dependent. In any time period, decisions are needed to be made on whether to provide the stimulating-demand fixed-expense promotion and how much product is needed. Wei and Chen (2011) considered a single item periodic-review system with a promotion scope of sale effort only. At the beginning of each period, the firm decides the size of order replenishment and sales effort jointly. They suggested the (s; S; z) policy; that is, the inventory is replenished based on an (s; S) policy, and the sales effort (z) is depending on the inventory level. Addition approach to model the joint promotion-inventory decision is the use of a super modular game approach (see Mesak et al (2015); Mesak et al (2016)).

The combination of the pricing, traditional advertising, and inventory control problems has gained popularity lately. Bing-Bing et al (2016) examined the problem in the newsvendor context with the use of a linear utility function and provided the instances when the loss aversions affect the optimal policy of order quantity, price, and advertising effort level and those when they do not. Zhang et al (2008), and Zhang (2012) consider the periodic review models which integrated decisions on pricing, promotion, and inventory with random demand. Both papers hold similar assumptions and models; but their later paper (Zhang, 2012) is more general and considers the broader possibilities such as whether the problem horizon is infinite, whether the fixed setup cost is included, and how events and decisions are sequenced. Their earlier model (Zhang et al 2008) is restrictive to the decision made in specific sequence: the first to decide whether to promote, and the next to use the optimal policy with specifications of two base-stock-list-price policies which correspond to whether or not a promotion is conducted. Their later work in 2012 claims the optimality of the quasi base-stock list-price target-promotion policy described by the existence of a critical inventory level, a list price and a target promotion. Sadigh et al (2016) analyzed the problem in a multi-echelon supply chain consisting of multiple suppliers, one manufacturer, and multiple retailers, using an iterative algorithm to find Nash Equilibrium point of the game. Chen (2015) used Stackelberg game theoretic framework in a two-echelon supply chain setting and gave an insight that the retailer and the manufacturer can affect the market size by investing in local advertising and making marketing expenditures on national brand names, but with diminishing returns.

This new model that is being presented here is different from the previous research mentioned above. Unlike the previous models, the new model takes into account the influence of social media on the demand level and it considers all three characteristics: the deterministic demand, infinite horizon model (EOQ-like), and the joint optimization of pricing, SM advertising (non-price promotion) and lot sizing. We assume that the effects of price and non-price advertising on demand are independent and addable: (1) the price affects the demand linearly or multiplicatively and the (2) SM advertising expense affects the demand positively with diminishing returns. Our assumptions are consistent with supply chain and marketing literatures and the empirical study relevant to social media power (such as Little, 1975; Whitin, 1955; Dada and Petruzzi, 1999; Asur and Huberman, 2010; Oh et al, 2017; Lassen et al, 2015; Hensel and Deis, 2010).

This paper is structured as follows: Section 2 introduces the advertising and inventory models and discusses their critical characteristics in both cases of price-exclusive and price-inclusive in the demand functions. Section 3 discusses the scenarios when price is not predetermined by the market. The business has room to decide the right price and advertising effort with the objective to maximize the profit margin after advertising expense; it is followed by the lot-sizing decision intended to minimize the cost of holding inventory and placing orders. A concluding remark and possible future research is in Section 4.

2. A MODEL FOR SM ADVERTISING AND LOT-SIZING WITH THE MARKET-PREDETERMINED PRICE

Three models have been analyzed. The first model does not include price in the demand function. This model will allow us to investigate the SM advertising investment effect toward lot-sizing decisions. Price is then introduced as one of the influential factors to the demand level in the second and the third models. The widely-used additive model (Whitin, 1955) is applied to describe the demand in the second model. We use multiplicative price-demand function, as introduced by the classic work of Karlin and Carr (1962) in the third model. No interactive effect is assumed between SM advertising efforts and the price towards the demand level.

The analysis throughout the paper is based on assumptions, some similar to Economic Order Quantity (EOQ) model i.e. the constant-demand infinite-horizon, no backlogging or lost sales are allowed in any period. The per-order setup cost, per-unit inventory holding cost and item-cost are all deterministic. By taking into account SM advertising investments, an annual cost therefore is the sum of the annual setup (or order) cost, inventory cost and advertising expense. The specifications of the functional forms will be illustrated later. The non-price advertising influences not only costs, but also demand levels, and thus the bottom-line corporate profitability. The objective of this study is to maximize annual profits by selecting appropriate values of decision variables of SM non-price advertising efforts (A), and lot-size (Q) for a pre-determined constant price (p).

2.1 *Model 2.1: the Basic Model*

The basic model excludes prices as an influential variable on demand. Later on, Models 2.2 and 2.3 include the impact of pricing on demand in the additive and multiplicative manners, respectively.

The relationship between demand and advertising expense was discussed in such papers as Little (1966), Lilien et al (1992), and Vidale and Wolfe (1957). Little (1966) used a quadratic sales (profits) function of promotion expense containing the parameter changed by a first-order autoregressive process. His control (adaptive) system is needed for its marketing decision designed to combine the new and old information and to determine the size of the experiment to be performed. Lilien et al (1992) showed several characteristics of sales functions depending on their advertising policy. As stated in several literatures, the steady state sales functions are S-shaped or concave to capture the diminishing return from advertising expense. Vidale and Wolfe (1957) proposed both dynamic sales and steady state concave sales functions based on the assumptions of sales decay and saturation and constant response parameters.

Traditionally organization determines a set budget amount, a marketer will decide how much to spend in various media channels to directly deliver messages to customers (Weinberg and Pehlivan E, 2011). In the social media, the advertising investment is to establish, build, or maintain relationships, generate positive word of mouth, engage conversations about products etc. If the social media marketers spend that same amount as the traditional marketing, it would result in others carrying or distributing their 'own' messages that are related to the marketer's brand. A number of papers show empirical evidence of strong positive correlations (relationship) between SM advertising investment and customer engagement and product sales. Therefore, our steady state sales function expresses the demand level increasing in SM advertising expenses, but at the diminishing rate of return due to sales decay and saturation and limited market size.

The specific functional form of the steady state sales in this study is $D = M(1- e^{-A/c})$ where D is the annual demand, M the sales saturation level, A the SM annual advertising investment, and the non-negative constant c as the function parameter. The low c illustrates the case of high demand even with minimum levels of SM advertising, consequently there is less potential new demand in this scenario. See **Figure 1**. Oppositely, the high c indicates the case of low demand with minimum SM advertisement, so additional SM advertisement leads to higher incremental demand prior to reaching sales saturation. The demand function is justified by its characteristics of increasing-concave, which represents the diminishing return from SM advertisement; the demand level is not allowed to exceed the sales saturate level (M); the new potential sales decreases as sales approach saturation. These characteristics are consistent with supply chain, marketing, and social media literatures (Urban, 1992; Lilien et al 1992; Vidale and Wolfe, 1957; Asur and Huberman, 2010; Oh et al, 2017; Lassen et al, 2015; Hensel and Deis, 2010).

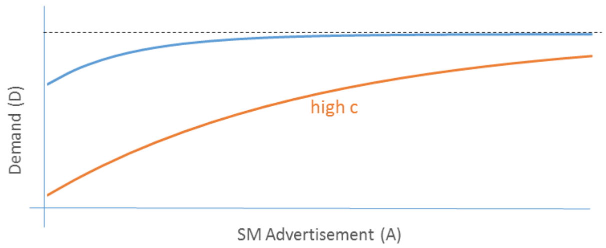

Figure 1 Demand function with varied values of parameter c

Thus, the annual profit, net of SM advertising expense, $\Pi(D, A, Q)$ has the specific functional form of

$$\Pi(D, Q, A) = Dmp - \frac{DS}{Q} - \frac{Qh}{2} - A,$$

where p is the unit price, m the profit margin in the percentage of p, S the cost of placing an order and h the per-unit annual holding cost, and Q the order quantity. The profit function incorporating the SM advertising effort on demand level and inventory expenses is illustrated by

$$\Pi(Q, A) = M(1-e^{-A/c})\left(mp - \frac{S}{Q}\right) - \frac{Qh}{2} - A \qquad (1)$$

Remark 1: $(mp - \frac{S}{Q}) > 0$ is necessary for the feasible solution with non-negative profit; that is, the order quantity must be large enough to allow the per-unit profit margin to cover the per-unit allocated ordering-cost.

Remark 2: $\Pi(Q, A)$ is defined on the convex-set domain G $= \{(Q, A): Q \geq 1 \text{ and } A \geq 0\} \subseteq \mathbb{R}^2$. $\Pi(Q, A)$ is second order partial differentiable and continuous.

Result 1: The profit function $\Pi(Q, A)$ is jointly concave in both A and Q.

Proof.

Recall the definition of a concave function of two variables: $f(x,y)$, defined on the convex set, is concave if the line segment joining any two points on the graph of f is never above the graph. Specifically, $\forall\, \lambda \in [0, 1]$, $f\,(\,(1-\lambda)(x, y)+\lambda(x', y')\,) \geq (1-\lambda)f(x, y) + \lambda f(x', y')$.

We apply this definition to our proof of concavity of $\Pi(Q, A)$ with the convex-set domain G = {(Q, A): $Q \geq 1$ and $A \geq 0$) $\subseteq \mathbb{R}^2$.

Suppose the function g(Q) is a single variable and concave on $Q \geq 1$; $\Pi(Q, A)$ in (1) a function of two variable is defined by $\Pi(Q, A) = g(Q)$. Note that every cross-section of the graph of Π parallel to the x-axis (representing values of Q) is the graph of function g. To prove the concavity of $\Pi(Q, A)$, we need to show that

$\Pi(\,(1-\lambda)(Q, A)+\lambda(Q', A')\,) \geq (1-\lambda)\Pi(Q, A) + \lambda\Pi(Q', A')$, $\forall\, \lambda \in [0, 1]$.

$$\Pi(\,(1-\lambda)(Q, A)+\lambda(Q', A')\,) = \Pi(\,(1-\lambda)Q+\lambda Q', (1-\lambda)A+\lambda A'\,) = g(\,(1-\lambda)Q+\lambda Q'\,)$$

Let a belong to the interval $[A, A']$ or $a = (1-\lambda)A+\lambda A'$

$$g(\,(1-\lambda)Q+\lambda Q'\,) = M(1-e^{-a/c})\left(mp - \frac{S}{(1-\lambda)Q+\lambda Q'}\right) - \frac{h((1-\lambda)Q+\lambda Q')}{2} - a$$

$$(1-\lambda)g(Q)+\lambda\, g(Q') = (1-\lambda)\left(M(1-e^{-a/c})\left(mp - \frac{S}{Q}\right) - \frac{hQ}{2} - a\right) + \lambda\left(M(1-e^{-a/c})\left(mp - \frac{S}{Q'}\right) - \frac{Q'h}{2} - a\right)$$

To show $g(\,(1-\lambda)Q+\lambda Q'\,) \geq (1-\lambda)g(Q)+\lambda\, g(Q')$, we need $\frac{1}{(1-\lambda)Q+\lambda Q'} \leq \frac{1-\lambda}{Q} + \frac{\lambda}{Q'}$

$\Leftrightarrow \frac{1}{(1-\lambda)Q+\lambda Q'} \leq \frac{1-\lambda}{Q} + \frac{\lambda}{Q'}$

$\Leftrightarrow QQ' \leq ((1-\lambda)Q'+\lambda Q) \bullet ((1-\lambda)Q+\lambda Q')$

$\Leftrightarrow \lambda(1-\lambda)(Q^2+Q'^2) - 2\lambda(1-\lambda)\,QQ' \geq 0$

$\Leftrightarrow (Q-Q')^2 \geq 0$

Therefore, $g(\,(1-\lambda)Q+\lambda Q'\,) \geq (1-\lambda)g(Q)+\lambda\, g(Q') = (1-\lambda)\Pi(Q, A) + \lambda\Pi(Q', A')$; that is,

$\Pi(\,(1-\lambda)(Q, A)+\lambda(Q', A')\,) \geq (1-\lambda)\Pi(Q, A) + \lambda\Pi(Q', A')$, $\forall\, \lambda \in [0, 1]$. We conclude that

$\Pi(Q, A)$ is concave on the convex domain G = {(Q, A): $Q \geq 1$ and $A \geq 0$) $\subseteq \mathbb{R}^2$.

Result 2: The profit-maximization solution, specified by the optimal annual investment in SM non-price advertisement together with the optimal quantity to order, denoted by A^{**} and Q^{**}, respectively, is achieved by applying the following three steps.

Step 1. Find Q^* from

$$Q^{*3} - \frac{S}{mp}Q^{*2} - \frac{2S}{h}\left(M - \frac{c}{mp}\right)Q^* + \frac{2MS^2}{hmp} = 0 \qquad (2)$$

Step 2. For each value of Q^*, compute A^* from

$$A^* = -c\ell n\left(\frac{cQ^*}{M(mpQ^*-S)}\right) \qquad (3)$$

No more than three pairs of (A^*, Q^*) will be found.

Step 3. (A^{**}, Q^{**}) is the (A^*, Q^*) which maximizes profit, as being illustrated by

$$(A^{**}, Q^{**}) = \underset{\forall (Q^*, A^*)}{\operatorname{argmax}}\, \Pi(Q, A)\,.$$

Proof.

Following Result 1: $\Pi(Q, A)$ is concave; the stationary points $\left(\frac{\partial\Pi(Q,A)}{\partial Q} = \frac{\partial\Pi(Q,A)}{\partial A} = 0\right)$ are global maximum points (A^*, Q^*).

$\frac{\partial\Pi(Q,A)}{\partial A} = -1 + \frac{M}{c}\left(mp - \frac{S}{Q}\right)e^{-A^*/c} = 0$. Thus, $e^{-A^*/c} = \frac{cQ}{MmpQ-MS}$

$\frac{\partial\Pi(Q,A)}{\partial Q} = \frac{-h}{2} + \frac{(1-e^{-A/c})MS}{Q^2} = 0.$

By substitute $e^{-A^*/c} = \frac{cQ}{MmpQ-MS}$ into $\frac{-h}{2} + \frac{(1-e^{-A/c})MS}{Q^2} = 0$, we get,

$$Q^{*3} - \frac{S}{mp}Q^{*2} - \frac{2S}{h}\left(M - \frac{c}{mp}\right)Q^* + \frac{2MS^2}{hmp} = 0$$

The value of Q^* from the above-mentioned equation is then entered to the following expression to find the value of

A*:

$$A^* = -c\ell n\left(\frac{cQ^*}{M(mpQ^*-S)}\right)$$

Thus, the profit maximization (A^{**}, Q^{**}) is found by entering the possible pairs of (A^*, Q^*) in to the profit function $\Pi(Q, A) = M(1\text{–}e^{-A/c})\,(mp - \frac{S}{Q}) - \frac{Qh}{2} - A$. The pair that provides the highest profit is the optimal solution (A^{**}, Q^{**}). This completes the proof.

Example 1. A company wants to determine its SM advertisement budget for the next year and schedule their production level accordingly. The price of their product will be \$1000 per unit (p = 1000) with the expected profit margin of 50% of total revenue (m = 50%). The saturated demand level is forecasted to be 10,000 units (M = 10,000). The plant manager has confirmed the future setup operation could cost \$ 600 each time (S = 600) and per-unit inventory holding cost historically is stable at \$50 per year (h = 50) and expected to continue, the constant parameter c is 850.

Our optimal procedures suggest:

- the two feasible Q^* of 1.20002 and 489.856; the other Q^* is not a real number;
- A^* for Q^* of 1.20002 is -16.9739. This suggests its infeasibility since the domain of feasible solution is defined on the space: $Q \geq 1 \times A \geq 0$.
- A^* for Q^* of 489.856 is 12,485.5503. Thus, the optimal solution is (A^{**} = \$12,485.5503, Q^{**} = 489.856 units). See **Figure 2**. This gives the total profit of \$ 4,963,017.47.

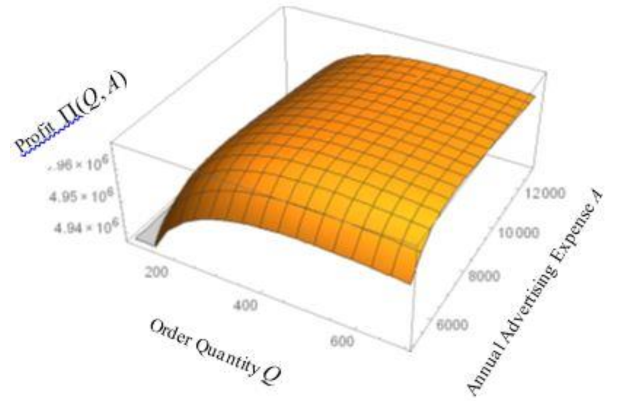

Figure 2 The three-dimensional graph of the profit function $\Pi(Q, A)$.

2.2 Model 2.2: A SM Advertising and Lot-sizing Model, with the Additive Price-Demand Relationship

Our second model has combined additive effects of price and SM advertising investment on the demand. Whitin's (1955) linear price-demand function is widely used and known to provide accurate prediction of sales for a finite horizon; it is so adapted to our model; thus, the specifications of the demand function is illustrated by total potential demand, $D(p, A) = \acute{M}(1\text{–}e^{-A/c}) + (a\text{–}bp)$.

As demonstrated by Whitin (1955), a is the intercept of his linear price-demand function, and b is its slope, with $p \leq \frac{a}{b}$, $a > 0$ and $b > 0$. If $\acute{M}$ is the maximum potential demand influenced by advertising, then $\acute{M} = M\text{–}(a\text{–}bp)$. This model allows the demand, uncaptured by price, to be reachable by the SM advertising efforts (i.e., reflected by the annual SM expenditure A). For a given price point,

$D(p, A) = (M - (a - bp))(1\text{–}e^{-A/c}) + (a\text{–}bp)$, and

$\Pi(Q, A) = [(M - (a - bp))(1\text{–}e^{-A/c}) + (a\text{–}bp)](mp - \frac{S}{Q}) - \frac{hQ}{2} - A$. That is,

$\Pi(Q, A) = [(M\text{–}D_p)(1\text{–}e^{-A/c}) + D_p](mp - \frac{S}{Q}) - \frac{hQ}{2} - A$, where $D_p = a\text{–}bp$. (4)

Remark 3: The impact of price on demand could be adjusted by letting $D_p = \alpha(a\text{–}bp)$; the higher non-negative constant α implies more portion of achievable demand (such as M) is affected by price. For simplicity, we assume $\alpha = 1$ throughout the paper; the results obtained from our analysis still hold true when α deviates from 1.

We found the similar results from Model 2.1 continue to exist for Model 2.2 as long as their order quantity is no fewer than $\hat{Q}$.

Result 3: $\Pi(Q, A)$ is jointly concave on its domain $G = \{(Q, A): Q \geq 1 \text{ and } A \geq 0\} \subseteq \mathbb{R}^2$ if and only if $Q \geq \hat{Q}$, where $\hat{Q} = \frac{S}{2mp}\left(-1 + \frac{2M}{M-(M-D_p)e^{-A/c}}\right)$.

Proof.

$\Pi(Q, A)$ is a twice-differentiable function of variables Q and A. To determine whether it is concave, we need to examine the Hessian of it.

$$\frac{\partial^2\Pi(Q,A)}{\partial Q^2} = -\frac{2(D_p+(1-e^{-\frac{A}{c}})(-D_p+M))S}{Q^3}$$

$$\frac{\partial^2\Pi(Q,A)}{\partial Q\partial A} = \frac{\partial^2\Pi(Q,A)}{\partial A\partial Q} = \frac{e^{-\frac{A}{c}}(-D_p+M)S}{cQ^2}$$

$$\frac{\partial^2\Pi(Q,A)}{\partial A^2} = -\frac{e^{-\frac{A}{c}}(-D_p+M)(mp-\frac{S}{Q})}{c^2}$$

$$H(Q,A) = \begin{pmatrix} -\frac{2(D_p+(1-e^{-\frac{A}{c}})(-D_p+M))S}{Q^3} & \frac{e^{-\frac{A}{c}}(-D_p+M)S}{cQ^2} \\ \frac{e^{-\frac{A}{c}}(-D_p+M)S}{cQ^2} & -\frac{e^{-\frac{A}{c}}(-D_p+M)(mp-\frac{S}{Q})}{c^2} \end{pmatrix}$$

$\Pi(Q, A)$ is concave if and only if H(Q, A) is negative semidefinite. If $H(Q, A)$ is negative definite, then $\Pi(Q, A)$ is strictly concave. Recall that Hessian is negative semidefinite if and only if $(-1)^k\Delta_k \geq 0$ for all principal minors. It is negative definite if and only if $(-1)^k D_k > 0$ for all leading principal minors. For the order one (k = 1) in our problem, $-\Delta_1 \geq 0$, $-D_1 > 0$: $-\Delta_1 = -D_1 = \frac{2(D_p+(1-e^{-\frac{A}{c}})(-D_p+M))S}{Q^3} > 0$, and $-\Delta_1 = \frac{e^{-\frac{A}{c}}(-D_p+M)(mp-\frac{S}{Q})}{c^2} > 0$.

For the order two (k = 2), $\Delta_2 \geq 0$, $D_2 > 0$:

$$\Delta_2 = D_2 = \left(-\frac{2(D_p+(1-e^{-\frac{A}{c}})(-D_p+M))S}{Q^3}\right) * \left(-\frac{e^{-\frac{A}{c}}(-D_p+M)\left(mp-\frac{S}{Q}\right)}{c^2}\right) - \left(\frac{e^{-\frac{A}{c}}(-D_p+M)S}{cQ^2}\right)^2.$$

Next check if the sign of Δ_2 or D_2 non-negative. $\Delta_2 \geq 0$ is equivalent to $\left(\frac{2(D_p+(1-e^{-\frac{A}{c}})(-D_p+M))S}{Q^3}\right) * \left(\frac{e^{-\frac{A}{c}}(-D_p+M)\left(mp-\frac{S}{Q}\right)}{c^2}\right) \geq \left(\frac{e^{-\frac{A}{c}}(-D_p+M)S}{cQ^2}\right)^2$

$\Leftrightarrow 2mp[(M-D_p)(1-e^{-A/c})+D_p] - \frac{2SM}{Q} + \frac{S}{Q}\left(M-D_p\right)e^{-A/c} \geq 0$

$\Leftrightarrow mp\left(M-(M-D_p)e^{-A/c}\right) - \frac{S}{Q}\left(M - \frac{(M-D_p)}{2}e^{-A/c}\right) \geq 0$

$\Leftrightarrow M\left(mp - \frac{S}{Q}\right) - (M-D_p)e^{-A/c}\left(mp - \frac{S}{2Q}\right) \geq 0$

$\Leftrightarrow\ 2\left(M-(M-D_p)e^{-\frac{A}{c}}\right)\left(mp-\frac{S}{Q}\right) \geq\ (M-D_p)e^{-A/c}\frac{S}{Q}$.

By observing the inequality above, LHS (Left-Hand-Side) is the twice of marginal profit (in monetary value such as $) and RHS (Right-Hand-Side) is the unmet saturated demand multiplied by the allocated setup cost per unit. LHS is increasing in Q where RHS is decreasing in Q.

Thus, there exists $\hat{Q}$ such that $2\left(M-(M-D_p)e^{-\frac{A}{c}}\right)\left(mp-\frac{S}{\hat{Q}}\right) =\ (M-D_p)e^{-A/c}\frac{S}{2\hat{Q}}$,

and $\hat{Q} = \frac{S}{2mp}\left(-1+\frac{2M}{M-(M-D_p)e^{-A/c}}\right)$.

Thus, $\Pi(Q, A)$ is concave ($\Delta_2 \geq 0$) if $Q \geq \hat{Q}$; $\Pi(Q, A)$ is strictly concave ($D_2 > 0$) if $Q > \hat{Q}$.

Following Result 3, the remaining analysis for Model 2.2 is made on the assumption of $Q > \hat{Q}$. We next state characteristics of the optimal solution denoted by the order quantity (Q**) and annual advertising expense (A**) in Result 4.

<u>Result 4</u>: The optimal solution (Q**, A**) are such that

$(A^{**},Q^{**}) = \underset{\forall\,(Q^*,A^*)}{\operatorname{argmax}}\,\Pi(Q,A)$ in (5) ,

where $Q^* \geq \hat{Q},\ Q^{*3} - \frac{S}{mp}Q^{*2} - \frac{2S}{h}\left(M - \frac{c}{mp}\right)Q^* + \frac{2MS^2}{hmp} = 0$, and

$A^* = -c\ell n\left(\frac{cQ^*}{(M-D_p)(mpQ^*-S)}\right)$.

Proof.

Since $\Pi(Q, A)$ is concave, the stationary points ($\frac{\partial\Pi(Q,A)}{\partial Q} = \frac{\partial\Pi(Q,A)}{\partial A} = 0$) are global maximum points (A*, Q*) as long as $Q^{**} \geq \hat{Q}$.

$\frac{\partial\Pi(Q,A)}{\partial A} = -1 + \frac{(M-D_p)e^{-\frac{A}{c}}(mp-\frac{S}{Q})}{c} = 0$. This give

$e^{-A^*/c} = \frac{cQ}{(M-D_p)(mpQ-S)}$

$\frac{\partial\Pi(Q,A)}{\partial Q} = -\frac{h}{2} + \frac{S}{Q^2}\left((M-D_p)\left(1-e^{-\frac{A}{c}}\right)+D_p\right) = 0.$

By substitute $e^{-A^*/c} = \frac{cQ}{(M-D)(mpQ-S)}$ into $-\frac{h}{2} + \frac{S}{Q^2}\left((M-D_p)\left(1-e^{-\frac{A}{c}}\right)+D_p\right) = 0$, we get,

$$Q^{*3} - \frac{S}{mp}Q^{*2} - \frac{2S}{h}\left(M-\frac{c}{mp}\right)Q^* + \frac{2MS^2}{hmp} = 0$$

The value of Q* is the order quantity that satisfy the above equation. Each of Q* (no more than three values of Q*) is then entered to the expression A* below.

$$A^* = \ -c\ell n\left(\frac{cQ^*}{(M-D_p)(mpQ^*-S)}\right)$$

Thus, the profit maximization (*A***, *Q***) is found by entering the possible pairs of (*A**, *Q**) in to the profit function $\Pi(Q, A) = [(M-D_p)(1-e^{-A/c}) + D_p](mp - \frac{S}{Q}) - \frac{hQ}{2} - A$, where $D_p = a-bp$. The pair that provides the highest profit is the optimal solution (*A***, *Q***).

Example 2. Suppose the company in example 1 has realized that part of the targeted demand (M) is influenced by price and only the remaining portion of it can be captured by SM advertising efforts. Assume linear-relationship between price and demand of $D_p = a-bp$, with a = 8,000 and b = 4. Thus, Dp = 4,000 units, with the remaining 6,000 units to be achieved the influence of SM advertisement. Assume all other problem parameters stay the same as what in example 1: p = 1000; m = 50%; M = 10,000; S = 600; h = 50; c = 850. The optimal decision is (A** = 6,941.47; Q** = 489.856), with $\hat{Q}$ = 0.6002. Note that there exists only one single pair of (Q*, A*) in the feasible domain of □□*Q*, *A*).

Figure 3 illustrates Results 3 and 4 in the graphical form.

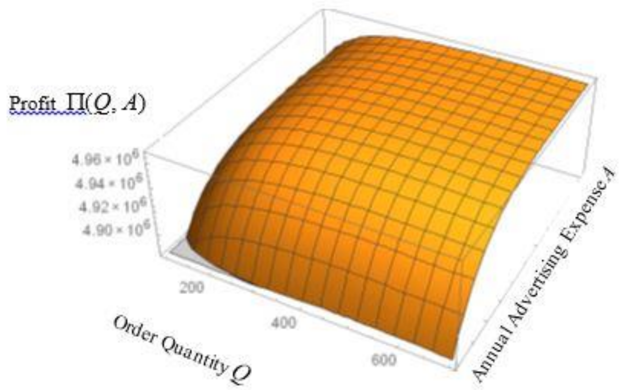

Figure 3 The three-dimensional graph of the profit function $\Pi(Q, A)$.

2.3 *Model 2.2: A SM Advertising and Lot-sizing Model, with the Additive Price-Demand Relationship*

The multiplicative relationship between price and demand was first initiated by Whitin (1955) and was later used by a number of relevant research (see: Carr (1962), Mill (1962), Data (1999), etc). The multiplicative model of price-demand relationship in steady state in our study is described by $D_p = ap^{-b}$, resulting in the new demand function $D(p, A) = (M-ap^{-b})(1-e^{-A/c}) + ap^{-b}$, and the corresponding profit function of

$\Pi\ (Q, A) = [(M-D_p)(1-e^{-A/c}) + D_p](mp - \frac{S}{Q}) - \frac{hQ}{2} - A$, where $D_p = ap^{-b}$. (5)

Remark 4: $\Pi\ (Q, A)$ in the multiplicative model (5) is similar to that of the additive model (4), except the way

D_p is defined differently. As a result, findings based on the additive model are applicable to the multiplicative model, again with the modifications in D_p functional expression.

2.4 Forms of Optimal Policy

All models in this section have accounted for the inventory policies of (1) equal lot-size, and (2) zero inventory property. Both are necessary conditions for optimality.

Optimal Policy 1: Zero Inventory Property

Zero-Inventory Property satisfies the optimal policy. Under Zero-Inventory Property, the beginning inventory is zero in the production/replenishment period.

Proof.

Consider a schedule with two consecutive setups in periods t and k where t < k. Suppose the beginning inventory in period k is r units. If we keep the schedule covering the entire problem horizon unchanged, reduce the batch size of production in period t by r units, and produce these r units in period k, we can reduce the holding cost of r-unit inventory without experiencing an increase in setup costs.

Optimal Policy 2: Equal Lot Size Property

Equal Lot-Size Property satisfies the optimal policy. Under Equal Lot-Size Property, the lot sizes are equal in all production/replenishment periods.

Proof.

Consider a schedule with two unequal lot sizes of Q, and Q′ where Q > Q′. Q units meet the demand for Q/D periods whereas Q′ units is enough to satisfy the demand for the length of Q′/D periods. Note that Q/D > Q′/D. If we reduce the lot size Q by one unit and increase the lot size Q′ by one unit, the amount of time spent in holding this one unit is reduced by Q/D − (Q′+1)/D and so is its inventory cost. Similarly, moving the second unit from lot size Q to batch size of Q′, we reduce the time of holding this unit by (Q-1)/D −(Q′+2)/D. By continuing the incremental analysis of transferring units from the bigger lot size of Q to the smaller lot Q′, we stop when both consecutive lot sizes become equal, that is (Q+Q′)/2, due to no more saving from moving inventory from the one lot to the next, or vice versa. Thus, equal lot sizes are necessary for cost-minimization or profit-maximization.

Section 3 is the scenario where price is not pre-determined.

3. THE MODEL FOR SM ADVERTISING, PRICING, AND LOT SIZING

3.1 The SM Advertising and Pricing Models

As known, business could run on a different set of constraints and sequential processes. In section 3, the target demand (or market share in item-units) was first determined from the marketing aspect, specifically how to select the right price point and to budget the SM advertise expense optimally, with the goal of maximizing the gross profit after the advertising expense (we refer to this type of profit as the operating profit throughout the paper); secondly, productions/order schedules are planed according to the demand with an intention to minimize the annual cost of ordering and inventory holding. We will revisit both additive and multiplicative price-demand relationships and apply them to Models 3.1 and 3.2, accordingly.

3.1.1 Model 3.1: A SM Advertising and Pricing Model, with the Additive Price-Demand Relationship

The operating profit function (6) is defined below and its characteristics are explored and discussed in Results 5 and 6.

$$\Psi(A, p) = [(M - (a - bp))(1 - e^{-A/c}) + (a - bp)]mp - A. \qquad (6)$$

Result 5: With the additive price-demand relationship, the operating profit $\Psi(A, p)$ is jointly concave in both A and p.

Proof.

The $f(x,y)$, defined on the convex set, is concave if $\forall\ \lambda \in [0, 1]$,

$$f\big((1-\lambda)(x, y)+\lambda(x', y')\big) \geq (1-\lambda)f(x,y) + \lambda f(x', y').$$

Suppose the function $g(p)$ is a single variable and concave on p. $\Psi(A, p)$ in (6) a function of two variable is defined by $\Psi(A, p) = g(p) = -bme^{-A/c}p^2 + [M - (M-a)e^{-A/c}]mp - A$. Note that every cross-section of the graph of Ψ parallel to the x-axis containing values of p is the graph of function g.

$$\Psi\big((1-\lambda)(A, p)+\lambda(A', p')\big) \geq (1-\lambda)\Psi(A, p) + \lambda\Psi(A', p'),\ \forall\ \lambda \in [0, 1].$$

$$\Psi\big((1-\lambda)(A, p)+\lambda(A', p')\big) = \Psi\big((1-\lambda)A+\lambda A',\ (1-\lambda)p+\lambda p'\big) = g\big((1-\lambda)p+\lambda p'\big)$$

Let a belong to the interval $[A, A']$ or $a = (1-\lambda)A+\lambda A'$

$$g\big((1-\lambda)p+\lambda p'\big) = -bme^{-A/c}((1-\lambda)p+\lambda p')^2 + [M - (M-a)e^{-A/c}]m((1-\lambda)p+\lambda p') - A$$

$$(1-\lambda)g(p)+\lambda g(p') = (1-\lambda)\left(-bme^{-\frac{A}{c}}p^2 + [M-(M-a)e^{-\frac{A}{c}}]mp - A\right) + \lambda\left(-bme^{-\frac{A}{c}}p^2 + [M-(M-a)e^{-\frac{A}{c}}]mp - A\right)$$

To prove the result, we need to show $g\big((1-\lambda)p+\lambda p'\big) \geq (1-\lambda)g(p)+\lambda g(p')$. It is equivalent to

$$(1-\lambda)p^2+\lambda p'^2 \geq ((1-\lambda)p+\lambda p')^2$$
$$\Leftrightarrow (1-\lambda)p^2+\lambda p'^2 \geq (1-\lambda)^2p^2 + \lambda(1-\lambda)2pp' + \lambda^2 p'^2$$
$$\Leftrightarrow (\lambda-\lambda^2)p^2+(\lambda-\lambda^2)p'^2 \geq \lambda(1-\lambda)2pp'$$
$$\Leftrightarrow p^2+p'^2 - 2pp' \geq 0$$
$$\Leftrightarrow (p-p')^2 \geq 0$$

Therefore, $g\big((1-\lambda)p+\lambda p'\big) \geq (1-\lambda)g(p)+\lambda g(p') = (1-\lambda)\Psi(A, p) + \lambda\Psi(A', p')$, that is,

$\Psi\big((1-\lambda)(A, p)+\lambda(A', p')\big) \geq (1-\lambda)\Psi(A, p) + \lambda\Psi(A', p')$, $\forall\ \lambda \in [0, 1]$. We conclude that

$\Pi(Q, A)$ is concave on its domain $G = \{(A, p): A \geq 0$ and $\frac{S}{Qm} \leq p \leq \frac{a}{b}\} \subseteq \mathbb{R}^2$.

Result 6: There exists the unique optimal pair of advertisement expense and price, denoted by (A*, p*) with the characteristics of

$p^* = \frac{2c}{am} - \frac{M}{b} + \frac{a}{b}$; $A^* = -c\ell n\left(\frac{c}{m(M-a)p^*+bmp^{*2}}\right)$.

Proof.

Following Result 5, the first order condition is used to derive the expression for p* and A*. That is,

$\frac{\partial\Psi(A,p)}{\partial A} = -1 + \frac{mpe^{-\frac{A}{c}}(M-a+bp)}{c} = 0$. This give $e^{-A^*/c} = \frac{c}{m(M-a)p+bmp^2}$.

$\frac{\partial\Psi(A,p)}{\partial p} = m(M-(M-a+2bp)e^{-\frac{A}{c}}) = 0$. This give $p^* = \frac{a}{2be^{-\frac{A}{c}}}$.

By substituting the expression for p* in to function $e^{-A^*/c}$, listed above, we get,

$p^* = \frac{2c}{am} - \frac{M}{b} + \frac{a}{b}$, and $A^* = -c\ell n\left(\frac{c}{m(M-a)p^*+bmp^{*2}}\right)$. This completes the proof.

We next analyze a similar problem with the change in demand-price relationship.

3.1.2 Model 3.2: A SM Advertising and Pricing model, with the Multiplicative Price-Demand Relationship

We consider the case when demand is multiplicative and assume $b < 1$. The operating profit is expressed by

$$\Psi(A, p) = [(M - ap^{-b})(1-e^{-A/c}) + ap^{-b}]mp - A \quad (7)$$

Its properties are demonstrated in Results 7 and 8.

Result 7: With the multiplicative price-demand relationship, $\Psi(A, p)$ is strictly concave on its domain $G = \{(A, p): A \geq 0$ and $\frac{S}{Qm} \leq p \leq (a(1-b^2))^{1/b} \subseteq \mathbb{R}^2\}$.

Proof.

We will prove the result by investigating the Hessian of $\Psi(A, p)$.

$$H(A,p) = \begin{pmatrix} -\frac{e^{-\frac{A}{c}}mp(M-ap^{-b})}{c^2} & \frac{e^{-\frac{A}{c}}mp^{-b}(a(-1+b)+Mp^b)}{c} \\ \frac{e^{-\frac{A}{c}}mp^{-b}(a(-1+b)+Mp^b)}{c} & a(-1+b)be^{-\frac{A}{c}}mp^{-1-b} \end{pmatrix}$$

We found that in the first order, $\Delta_1 = D_1 = -\frac{e^{-\frac{A}{c}}mp(M-ap^{-b})}{c^2} < 0$, and $\Delta_1 = a(-1+b)be^{-\frac{A}{c}}mp^{-1-b} < 0$. For the order two (k = 2), $\Delta_2 \geq 0$ indicates concave of the function or $D_2 > 0$ indicates the function is strictly concave. Note $\Delta_2 = D_2$.

$$\Delta_2 = D_2 = \left(-\frac{e^{-\frac{A}{c}}mp(M-ap^{-b})}{c^2}\right) * \left(a(-1+b)be^{-\frac{A}{c}}mp^{-1-b}\right) - \left(\frac{e^{-\frac{A}{c}}mp^{-b}(a(-1+b)+Mp^b)}{c}\right)^2.$$

We will need to investigate whether it is non-negative; that is to check

$ab(b-1)(M-ap^{-b}) - p^{-b}(a(b-1) + Mp^b)^2 \geq 0$

$\Leftrightarrow ab(b-1)(M-ap^{-b}) \geq p^{-b}(a(b-1) + Mp^b)^2 = p^{-b}(a^2(b-1)^2 + M^2p^{2b}) + 2a(b-1)M$

Let us replace ap^{-b} by D in the above function. Note that $(b-1) < 0$. We then get

$\Leftrightarrow ab(b-1)(M-D) \geq a(b-1)^2D + Ma/D + 2a(b-1)M$

$\Leftrightarrow ab(M-D) \leq a(b-1)D + \frac{Ma}{D(b-1)} + 2aM$

$\Leftrightarrow ab(M-2D) \leq -aD + \frac{Ma}{D(b-1)} + 2aM$

$\Leftrightarrow b(M-2D) \leq -D + \frac{M}{D(b-1)} + 2M$

$\Leftrightarrow D(1-b) - bD \leq M(1-b) + \frac{M}{D(b-1)} + M$

Clearly $D(1-b) \leq M(1-b)$. Next we will show that $-bD < M(1+\frac{1}{D(b-1)})$ by showing that $-bD \leq D(1+\frac{1}{D(b-1)})$ iff $p \leq (a(1-b^2))^{1/b}$. Note that $D(1+\frac{1}{D(b-1)}) < M(1+\frac{1}{D(b-1)})$.

$-bD \leq D(1+\frac{1}{D(b-1)})$ $\Leftrightarrow \frac{1}{(1-b)} \leq D(1+b)$

$\Leftrightarrow D(1-b^2) \geq 1$ $\Leftrightarrow ap^{-b}(1-b^2) \geq 1$

$\Leftrightarrow p \leq (a(1-b^2))^{1/b}$

So, $D_2 > 0$, and $\Psi(A, p)$ is strictly concave in A and p.

Result 8: There is an optimal pair of advertisement expense and price, denoted by (A*, p*) with the following characteristics of

(1) p* has to satisfy $mM^2p^{b+1} - mMap - cMp^b + a(1-b)c = 0$

(2) $A^* = -c\ell n\left(\frac{c}{mp(M-ap^{*-b})}\right)$

Proof.

Following Result 7, the stationary points ($\frac{\partial\Psi(A,p)}{\partial A} = \frac{\partial\Psi(A,p)}{\partial p} = 0$) are global maximum for the operating profit $\Psi(A, p)$. First order condition is an optimal way to derive the expression for p* and A*.

$\frac{\partial\Psi(A,p)}{\partial A} = -1 + \frac{e^{-\frac{A}{c}}mp(M-ap^{-b})}{c} = 0$. This give $e^{-A^*/c} = \frac{c}{mp(M-ap^{*-b})}$.

This is equivalent to expression of A* in Result 8.

Next, we want to find p*.

$$\frac{\partial\Psi(A,p)}{\partial p} = M(1-e^{-A^*/c}) + a(1-b)e^{-A^*/c}p^{*-b} = 0$$

$$\Leftrightarrow e^{-A^*/c} = \frac{M}{M-a(1-b)p^{*-b}}$$

Next we substitute $e^{-A^*/c} = \frac{c}{mp(M-ap^{*-b})}$ to the left-hand-side of the equation above. We get

$$\frac{c}{mp(M-ap^{*-b})} = \frac{M}{M-a(1-b)p^{*-b}}$$

$\Leftrightarrow cM - a(1-b)cp^{*-b} = mM^2p^* - mMap^{*1-b}$

$\Leftrightarrow mM^2p^{*b+1} - mMap^* - cMp^{*b} + a(1-b)c = 0$, which is the equation to be satisfied to obtain p*

3.2 The Lot-Sizing Model

Once the optimal p* and A* is found by using by Results 6 and 8. The next step is to find the demand level D* that maximizes the operations profit. Specifically,

$D^* = (M-(a-bp^*))(1-e^{-A^*/c}) + a\text{-}bp^*$ for Model 3.1 (8)

$D^* = (M-ap^{-b})(1-e^{-A/c}) + ap^{-b}$ for Model 3.2 (9)

Then the optimal order quantity that minimizes the annual cost of placing orders and holding inventory is, in fact, the classical model EOQ, that is,

$$Q^* = \sqrt{\frac{2D^*S}{h}}.$$

We have derived the critical results mathematically for five different models of SM advertising, pricing, and lot sizing. Our scope of analysis has been how to optimally find the appropriate SM advertising efforts, the right price, and the cost-minimum lot size. We showed the models that are jointly concave among the decision variables and presented the procedures to obtain the optimal schedule together with the fundamental form of the optimal production policy. We will next provide the insightful discussion relevant to structural property of decision variable and the optimization programing.

4. INSIGHT DISCUSSIONS

The problem could be modeled as programs with objective functions and constraints, as listed in **Table 1**. Results 1-8 in fact provide clear and specific objectives and constraint expressions. Although there is mathematical software available to solve for optimal solutions, a number of them are applicable only to the linear functions or non-linear functions with limitations (such as least-square, quadratic, general algebra, etc.) Our program features explicit tight integration of exponential and quadratic specifications interacting with decision variables, consequently, complicating the programs. Some software does not guarantee to provide for global optimization.

Table 1 Optimization programs at the interface of operations with social media marketing

Models	
Basic Model	**Obj:** $\max_{Q\geq 1, A\geq 0} M(1{-}e^{-\frac{A}{c}})\,(mp - \frac{S}{Q}) - \frac{Qh}{2} - A$ **St:** $Q^3 - \frac{S}{mp}Q^2 - \frac{2S}{h}\left(M - \frac{c}{mp}\right)Q + \frac{2MS^2}{hmp} = 0;$ $A + c\ell n\left(\frac{cQ}{M(mpQ-S)}\right) = 0.$
A SM Advertising and Lot-sizing Model, with Exogenous Price-Demand Additive Relationship	**Obj:** $\max_{A\geq 0, Q\geq 0} [(\text{M}-\ (a-bp))(1-e^{-A/c})\ +\ a-bp](mp - \frac{S}{Q}) - \frac{hQ}{2} - A$ **St:** $Q \geq \frac{S}{2mp}\left(-1 + \frac{2M}{M-(M-(a-bp))e^{-A/c}}\right);$ $Q^3 - \frac{S}{mp}Q^2 - \frac{2S}{h}\left(M - \frac{c}{mp}\right)Q + \frac{2MS^2}{hmp} = 0;$ $\text{A} + c\ell n\left(\frac{cQ}{(M-(a-bp))(mpQ-S)}\right) = 0.$
A SM Advertising and Lot-sizing Model, with Exogenous Price-Demand Multiplicative Relationship	**Obj:** $\max_{A\geq 0, Q\geq 0} [(\text{M}-\ ap^{-b})(1-e^{-A/c})\ + ap^{-b}](mp - \frac{S}{Q}) - \frac{hQ}{2} - A$ **St:** $Q \geq \frac{S}{2mp}\left(-1 + \frac{2M}{M-(M-ap^{-b})e^{-A/c}}\right)\ ;$ $Q^3 - \frac{S}{mp}Q^2 - \frac{2S}{h}\left(M - \frac{c}{mp}\right)Q + \frac{2MS^2}{hmp} = 0;$ $\text{A} + c\ell n\left(\frac{cQ}{(M-ap^{-b})(mpQ-S)}\right) = 0.$
A SM Advertising and Pricing Model, with Endogenous-Price Demand Additive Relationship	**Obj:** $\max_{A\geq 0, p\geq 0} [(M-(a-bp))(1-\text{e}^{-A/c})\ +\ (a-bp)]mp - A$ **St:** $A + c\ell n\left(\frac{c}{m(M-a)p+bmp^2}\right) = 0;$ $p - \frac{2c}{am} + \frac{M}{b} - \frac{a}{b} = 0.$
A SM Advertising and Pricing model, with the Endogenous-Price Demand Multiplicative Relationship	**Obj:** $\max_{A\geq 0, p\geq 0}\ [(M - ap^{-b})(1\text{–}e^{-A/c})\ +\ ap - b]mp - A$ **St:** $p - (a(1-b^2))^{1/b} \leq 0;$ $p - \frac{S}{Qm} \geq 0;$ $mMp^{b+1} - map - cp^b + \frac{a(1-\text{b})\text{c}}{M} = 0;$ $\text{A} + c\ell n\left(\frac{c}{mp(M-ap^{-b})}\right) = 0.$

The structural properties of decision variables shows their interesting relationships. For instance, at the optimum, solution for social media investment goes up with price. This is independent of whether price is exogenous or endogenous and whether the price and demand are related in the additive or multiplicative manner. If price increases, their negative relationship generates a decreasing demand. This leaves more demand untapped by price available for social media advertisement. Further justification is the influence of higher per-unit marginal profit due to price increase. It allows business to spend more money on advertising to convince the customers to pay a premium. Since price and demand are interactive factors, one could expect the vise-versa effect of advertising and price. A quadratic characteristic with relationships of lot sizing and other decision variables is non-trivial.

We will close this presentation by providing concluding remarks and direction for future research.

5. CONCLUDING REMARK

With the wide spread of social media and respective consumer engagement behavior, several industries and corporates (such as Ford Motor Company, McDonald, movies and fashion industries) employ their social media to promote / advertise their products. Traditionally, a marketer will decide how much to spend in various media channels to directly deliver messages to customers; in the social media era, the business invests in social media to establish, build, or maintain relationships, generate positive word of mouth and engage conversations about the product. This results in others to carry or distribute the messages related to the business brand. We found the limited number of research that applied mathematical modeling relevant to social media advertisement and sales.

Our work contributes by addressing the mathematic deterministic model to incorporate the influence of social media advertising onto the demand level. The models presented in this paper can provide the optimal decisions at the interface between operations (lot-sizing) and marketing (pricing and social media advertising investment). Several different scenarios are considered. When price is pre-determined by the market with the model decision variables of how much and to inventory and how much to budget for social media expenses. If the product price becomes part of the decision variable together with other variables including lot-sizes and social media advertising budget. Our analysis includes both the additive and multiplicative price-demand functions which have been long well known and widely used in various research work.

We found the fundamental characteristics of profit functions for both scenarios mentioned above. Specifically, when the price is predetermined by the market, the profit function is jointly concave in the social media advertising expense and lot size; but when price is part of the business decision, the profit after social media expense is jointly concave in the social media expense and price. The optimal algorithm is proposed, and forms of the optimal production policy are discussed in this context. The concave characteristic of profit function, in fact, guarantees the uniqueness of the optimal solutions, and more importantly if any are at all possible.

An interesting future research is to extend the models of social media advertising, lot sizing, and pricing to the stochastic-demand continuous-review model (Q-R policy) into the problems with finite or infinite horizon. Our work has left rooms for the periodic-review inventory policy to be modeled mathematically. One possible empirical study is to create the demand functions best-fitting with the social media expense by industries or business segments. Another extension is the application of dynamic programming to the problems, in which the demand as a function of social media advertising effort and price is dynamic.

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