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A Hybrid Forecasting Technique to Deal with Heteroskedastic Demand in a Supply Chain

Sanjita Jaipuria
Indian Institute of Management, Mayurbhanj Complex, Nongthymmai
Shillong – 793 014 East Khasi Hills District, Meghalaya, India
Email: sanjita.jaipuria@gmail.com *(Corresponding Author)*

S. S. Mahapatra
Department of Mechanical Engineering
National Institute of Technology Rourkela, 769008 India
Email: mahapatrass2003@gmail.com

ABSTRACT

Under demand uncertain environment, maintaining a proper safety stock is very important to cope with the stock-out situation. Improper estimation of safety stock quantity leads to an improper estimation of the order and further causes bullwhip effect and net-stock amplification. In practice, demand is heteroskedastic in nature i.e. the variance of the demand varies with time. Therefore, it is important to predict the changing demand variance to update safety stock level in each replenishment cycle. The Autoregressive Integrated Moving Average (ARIMA) model applied to predict the mean demand assuming it is homoscedastic in nature. Whereas, the Generalized Autoregressive Conditional Heteroskedasticity (GARCH) model deal with heteroskedastic demand and help in projecting the changing demand variance. Hence, a combined approach of ARIMA and GARCH (ARIMA-GARCH) model has been proposed to evaluate the safety stock level and order quantity. The performance of ARIMA and ARIMA-GARCH has been evaluated considering the demand from a cement manufacturing company. The cement demand is seasonal in pattern and highly fluctuate. Using cement demand data, ARIMA (2, 1, 1) (0, 1, 1)$_{12}$ and GARCH (2, 1) model is identified to forecast 12-months ahead mean and variance of demand to determine the safety stock and order quantity in each replenishment cycle applying the equations proposed by Zhang (2004) and Luong & Phien (2007). Further, bullwhip effect and net-stock amplification ratio are estimated to evaluate the performance of ARIMA-GARCH model against the ARIMA model. From the study, it has found that ARIMA-GARCH model outperforms the ARIMA as it updates the safety stock to calculate order quantity in each replenishment cycle.

Keywords: autoregressive integrated moving average; bullwhip effect; generalized autoregressive conditional heteroskedasticity; net-stock amplification

1. INTRODUCTION

The bullwhip effect (BWE) can be described as variation in order in relation to the variant in demand and is one of the most important concern in a supply chain. Chopra *et al.* (2006) have stated that BWE leads to increase in total cost due to rise in various costs such as production, inventory carrying, shipping and receiving cost and decrease in service level and profitability. BWE within a supply chain can be reduced by controlling the causes like non-zero lead-time, order batching, price fluctuations, supply shortages and demand forecasting as proved by Lee *et al*. (1997). In non-deterministic demand case, order quantities are calculated using predicted demand and hence accuracy of estimated order quantity depends on forecasting accuracy. Therefore, forecasting of demand plays a major role to control the BWE and there are different time series models have been suggested and evaluated by many authors. These models are: moving average (MA) model (Chen *et al*., 2000a; Hong and Ping, 2007), exponential moving average (EMA) model (Chen *et al*., 2000b), exponentially weighted moving average (EWMA) model (Hong and Ping, 2007), first-order autoregressive (AR(1)) (Luong, 2007), higher order of AR i.e. AR(p) (Luong and Phien, 2007), autoregressive moving average (ARMA) model (Zhang, 2004; Duc *et al*., 2008a; 2008b) and autoregressive integrated moving average (ARIMA) model (Gilbert, 2005; Gilbert and Chatpattananan, 2006). These models have demonstrated that BWE can be moderated by monitoring AR and MA coefficient and lead-time. Practically, it is hard to control these parameters. These models are applicable for the data series that is stationary and homoscedastic in nature i.e., it is assumes that demand variance remains constant throughout the considered time horizon.

However, in reality demand series is heteroskedastic in nature. Therefore, it is difficult to capture the fluctuating demand variance using AR, MA, ARMA or ARIMA model. In order, to cope with demand variation, a safety stock is kept in each stage of a supply chain. Safety stock serves as a buffer in of anticipation to the stock-out situation when demand is uncertain. In the order up-to level inventory control policy the order quantity calculation go wrong if the safety stock is not accurately estimated. Hence, an accurate calculation of safety stock is essential to manage inventory and demand. Accurate forecasting of demand variance is required to recalculate the safety stock quantity in each replenishment period and accordingly the order quantities are calculated. To predict the demand variance the time series forecasting model called generalized autoregressive conditional heteroskedasticity (GARCH) model can be applied (Liang, 2013; Hor *et al*., 2006; Zhang, 2007).

There is no adequate literature exist those have highlighted the procedure to update the safety stock level and order quantity estimation in each replenishment cycle following the order up-to level inventory control policy under heteroskedastic demand environment. Hence, to solve the constraint associated with ARIMA model to calculate the safety stock and order quantity at each replenishment period a combined approach of ARIMA and GARCH model denoted as ARIMA-GARCH, has proposed in this work. The influence of BWE can be visualized in production switching cost or ordering cost. Due to variation in order quantity at the demand side, a high safety stock level has to hold at the supply side of supply chain and this leads to high inventory holding cost. Therefore, it is not only needed to measure the variation in order quantity but also to quantify the variation in stock level. Fluctuation in inventory level can be represent through net-stock amplification (Boute and Lambrecht, 2009). In this, study BWE and net-stock amplification have considered as performance measures to examine the performance of ARIMA-GARCH model following the base-stock inventory control policy. The performance of the proposed ARIMA-GARCH model has been described through a case study.

2. LITERATURE REVIEW

The time series ARIMA model has been applied in different areas of research to forecast future value. Berthouex and Box (1996) applied the EWMA model to predict 1-5days ahead performance of the water treatment plant so that the operators can take preventive action. Tse (1997) has used the ARIMA model to forecast the real estate market price in Hong Kong. Saab *et al*. (2001) applied AR, ARIMA and AR (1) combined with high-pass filter to predict one-step ahead month electricity consumption in Lebanon. Kumar and Jain (1999) have used the time series ARIMA model to predict traffic noise. Chavez *et al*. (1999) have applied the ARIMA model to forecast the future production and consumption rate of energy in Asturias. Ho and Xie (1998) have studied the use of time series modelling in the repairable system to analyze and forecast the failure interval for a mechanical system to test the reliability of the system. To interpret the relationship between energy demand and economic status, Ediger and Akar (2007) have used ARIMA and seasonal ARIMA to predict the energy demand in Turkey from year 2005 to 2020. Using 42 years (from 1960 to 2007) of monthly discharge data, Valipour *et al*. (2013) predicted the inflow rate of the Dez dam reservoir applying ARMA, ARIMA and static and dynamic artificial neural network model and compared their performance. Erdogdu (2007) projected the future growth of electricity demand in Turkey using ARIMA modelling. Rekhi *et al*. (2020) applied ARIMA model to forecast the air quality at Delhi, India. 15. Dritsakis and Klazoglou (2019) used ARIMA model to forecast the health expenditure in USA.

Wang and Yeh (2014) applied web-based decision support system architecture for more accurate and effective forecasting. Kavasseri and Seetharaman (2009) have predicted the one-day and two-days ahead wind speed using the fractional ARIMA model. Hossain *et al*. (2006) have used the ARIMA model to make three types of forecasts based on historical, ex-post and ex-ante data for predicting commodity prices in Bangladesh. Zhou *et al*. (2006) have applied the ARIMA approach to project the hourly market-clearing price in electricity spot markets with error correction and confidence interval estimation. Bullwhip effect is one of the major issue in supply chain. Shee and Kaswi (2015) have analysed the effect of ordering behavior on bullwhip effect in a three-echelon multinational and local supply chain in Indonesia. Lee *et al*. (1997) have stated that inaccurate forecasting is one of the major reasons of BWE in a supply chain. Therefore, to monitor it throughout the supply chain ARIMA models has been used and many authors have highlighted it. Dhahri and Chabchoub (2007) have proposed the nonlinear goal programming model based on ARIMA process to moderate BWE within a supply chain. The influence of lead-time and information sharing on the on-hand inventory in a two-echelon serial supply chain is analyzed by Agrawal *et al*. (2009) assuming customer demand follows AR process at the retailer's end. Considering ARIMA (0, 1, 1) demand process at a retail store Babai *et al*. (2013) have analyzed the relationship between the precision of forecasting and inventory performance. A combined approach of ARIMA and neural network has used by Aburto and Weber (2007) for demand forecasting. Duc *et al*. (2008) have applied time series ARMA (1, 1) model to test the influence of AR and MA coefficient and lead-time on BWE. From the study, they have proved that BWE occurs in the case where the AR coefficient higher than the MA coefficient. For a multistage supply chain, Gilbert (2005) has recommended a general expression to compute BWE when demand is predicted using ARIMA process and verified that the BWE becomes high in the case when lead-time is lengthy and demand is autocorrelated. Boute and Lambrecht (2009) have examined the predictive models such as moving average, exponential smoothing and minimum mean square error for a two-stage supply chain that followed order up-to level inventory replenishment policy. They reported that controlling the BWE does not necessarily control the inventory carrying cost. Inventory carrying cost is significantly influenced by keeping high safety stock due to fluctuation in inventory.

The time series ARIMA model assumes that demand variance remains constant throughout the forecasting period and hence difficult to predict. However, variation in demand variance can be predicted using the GARCH model. Engle (1982) have introduced the Autoregressive Conditional Heteroskedasticity (ARCH) model to estimate the means and variances of the inflation rate. He has been described that ARCH model implements autoregressive structure on the conditional variance of the process error to explain its time-varying change. Further, Bollerslev (1986) extended the ARCH model by placing the ARMA structure on the conditional variance of the process error and the model popularized as GARCH model. Liang (2013) have applied GARCH model to examine and predict the field failure data of the repairable system. Hor *et al.* (2006) have forecasted daily electricity load using ARIMA model and forecasted the maximum demand using the GARCH process. Simonato (2019) has applied GARCH model considering non-normal innovation to determine the American option prices. Trapero *et al*. (2019) applied kernel density estimation and GARCH (1, 1) to calculate the safety stock. Further, they proved that kernel density estimation is suitable for shorter lead-time whereas for long lead GARCH model is suitable to predict demand variance and determine the safety stock level. Zhang (2007) have applied the AR (1) model to characterize the dynamic changes in demand level over time and GARCH (1, 1) model to define the changing demand variance. Choudhry *et al*. (2019) applied seven variants of GARCH model to forecasts the hedge ratios of emerging European stock futures markets such as Greece, Hungary, Poland and the UK Using this setting, the influence of temporal heterogeneous on the inventory performance is quantified considering order up-to level inventory control policy. It is also shown that heteroskedasticity on the forecasting precision can be additively decomposed from the total forecasting error variance.

3. TIME SERIES FORECASTING MODEL

The time series demand-forecasting model can be express as follows:

$$D_{t+1} = f_\theta\left(D_t + D_{t-1} + \ldots + D_{t-N+1}\right) \tag{1}$$

where, D_{t+1} is the demand value in time t+1 that to be determined, this is forecasted using the current and past value i.e. existing demand data. The past demand data can be applied to estimate the mean and variance of future demand. The following paragraph briefly describes the time series ARIMA and GARCH model those are applicable to forecast mean demand and variance of demand respectively.

3.1 The ARIMA Model

For forecasting using the ARIMA model is presumed that the demand series is stationary and homoscedastic in nature. Five general steps such as (i) data preparation (ii) model selection (iii) estimation (iv) diagnostic checking and (v) forecasting are involved to identify ARIMA model and make prediction (Makridakis *et al*., 1998). The ARIMA model for non-seasonal demand is express in the form of ARIMA (p, d, q) and for the seasonal demand, the model is expressed as ARIMA (p, d, q) $(P,D,Q)_s$. The Equation (2) describes the ARIMA(1,1,1) model whereas Equation (3) describes the ARIMA(1,1,1)$(1,1,1)_{12}$ the seasonal model (Box and Jenkins, 1976; Box *et al*. 1994; Box *et al*. 2015).

$$Y_t = c + (1+\phi_1)D_{t-1} - \phi_1 D_{t-2} + e_t - \theta_1 e_{t-1} \tag{2}$$

$$\begin{aligned} Y_t = {} & c + (1+\phi_1)D_{t-1} - \phi_1 D_{t-2} + (1+\Phi_1)D_{t-12} - (1+\phi_1+\Phi_1+\phi_1\Phi_1)D_{t-13} + (\phi_1+\phi_1\Phi_1) \\ & D_{t-14} - \Phi_1 D_{t-24} + (\Phi_1+\phi_1\Phi_1)D_{t-25} - \phi_1\Phi_1 D_{t-26} + e_t - \theta_1 e_{t-1} - \Theta_1 e_{t-12} + \theta_1\Theta_1 e_{t-13} \end{aligned} \tag{3}$$

In the above equations, term p and q represent the non-seasonal order of the AR and MA part respectively whereas, P and Q represent the seasonal AR and MA order respectively. 's' represents number of seasonal periods, c is the constant term, ϕ_j and Φ_j are non-seasonal and seasonal j[th] AR parameters, θ_j and Θ_j are non-seasonal and seasonal j[th] MA parameters, d and D non-seasonal and seasonal degree of differencing involved, e_t, e_{t-p} and e_{t-q} are the error term at time t, t-p and t-q respectively.

To identify the ARIMA model the software STATISTICA 9 has been used in this work. Following paragraph, brief the GARCH model.

3.2 The GARCH Model

The time series ARCH model relaxes the fundamental assumption of the ARIMA model and it has the competency to apprehend the fluctuating financial time series data. The Equation (4) and Equation (5) represent the ARCH model of order q denoted as ARCH (q) (Engle, 1982).

$$r_t = \sigma_t \varepsilon_t, \text{ where } \varepsilon_t \sim N(0,1) \tag{4}$$

$$\sigma_t^2 = \alpha_0 + \alpha_1 r_{t-1}^2 + \ldots + \alpha_q r_{t-q}^2 \tag{5}$$

The time series ARCH model extended to the GARCH model. The Equation (6) is the mathematical expression of GARCH (p, q) demand process with restriction (Bollerslev, 1986). This equation states that the conditional variance of the error is associated with both the squares of past error and past conditional variance.

$$\begin{aligned} & \sigma_t^2 = \alpha_0 + \alpha_1 r_{t-1}^2 + \ldots + \alpha_q r_{t-q}^2 + \beta_1 \sigma_{t-1}^2 + \ldots + \beta_p \sigma_{t-p}^2 \\ & \alpha_i \ge 0,\ i = 1,2,\ldots,q,\ \beta_j \ge 0,\ j = 1,2,\ldots,p \text{ and } \left|\sum \alpha_i + \sum \beta_j\right| < 1. \end{aligned} \tag{6}$$

where $\{r_t\}$ denotes the mean correlated return, the past conditional variance is σ_{t-p}^2, ε_t is the white noise and p and q are the positive integers demonstrating the order of GARCH process, t > max (p, q).

The GARCH time series model is applies to the return series. Different statistical procedures are followed to identify the GARCH model for the prediction of demand variance. These are: (i) conversion of data series into return series (ii) test for existence of ARCH effect and serial correlation in return series (iii) estimation and analysis of model (iv) comparative analysis for fitted model (v) post-diagnostic checking for the fitted model and (vi) forecasting using identified model. In this study, MATLAB 2013 software has been used to identify the GARCH model.

4. THE PROPOSED FORECASTING MODEL

Safety stock is kept by an organization to manage the stock-out situation occur due to uncertainty demand and supply. Demand variance is one of the key parameters to estimate the safety stock. In the case of heteroskedastic demand series, prediction of changing demand variance is crucial for the accurate calculation of the safety stock and order quantity at each inventory replenishment period. The changing demand variance is difficult to predict since the ARIMA model can handle only the homoscedastic data series. However, this limitation of the ARIMA model can be resolved by GARCH model. Therefore, the ARIMA model is integrated with the GARCH model represented as ARIMA-GARCH to enhance the performance of forecasting model. The ARIMA-GARCH model is shown in **Figure 1**.

The mean demand is predicted using the ARIMA and the demand variance is predicted through the GARCH model. Further, the mean and variance of the demand have used to calculate the safety stock quantity and order quantity.

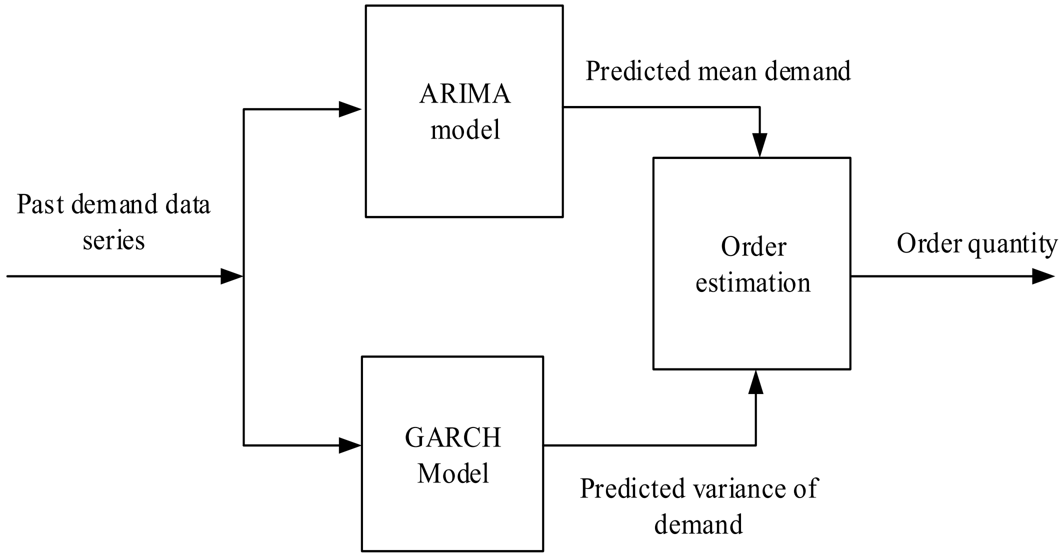

Figure 1 The structure of ARIMA-GARCH model

5. CASE STUDY

A cement manufacturing company XYZ Pvt. Ltd. located in eastern part of India is considered as a case in this study. To examine the performance of the ARIMA and ARIMA-GARCH model six years monthly demand data from April 2006 to March 2013 are collected from the company. The statistical procedure described in section 3.1 are followed to identify the ARIMA model. The **Figure 2** describes the time series plot for the collected cement demand.

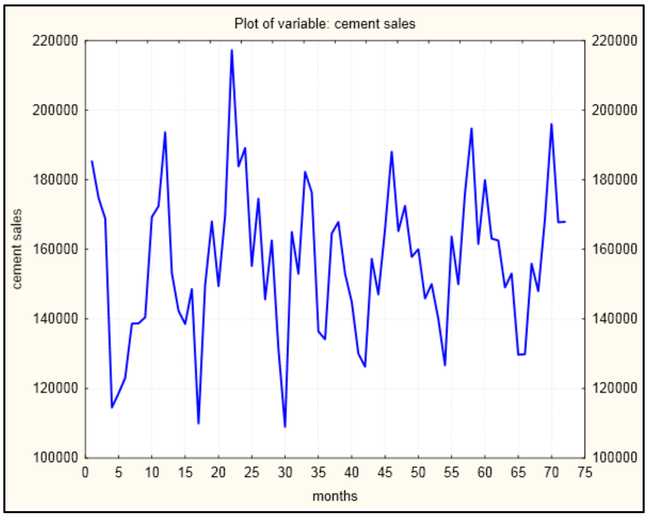

Figure 2 Time series plot for cement demand

To verify the presence of stationarity in cement demand data the Augmented Dickey-Fuller (ADF) test is conducted at significance level of $0.05 (\alpha = 5\%)$. The test determined that h=0 and p-value=0.3906 indicates that it failed to reject the null hypothesis and hence cement demand is non-stationary in nature. The **Figure 3** and **Figure 4** describes the autocorrelation function (ACF) and partial autocorrelation function (PACF) plots respectively for the demand series conducted for 18-lag period. In Figure 3, the significant spike observed at lag 12 signifies that demand series is seasonal in nature. Therefore, the demand data is transformed into stationary form by differencing with lag 12 and hence D=1.

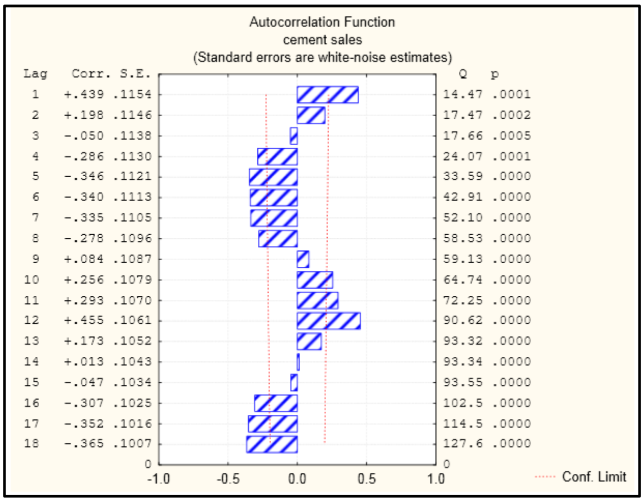

Figure 3 ACF plot of cement demand

Then the resultant series is again differenced with lag 1 and hence d =1. The time series plot **(Figure 5)** of the resultant cement demand after transformation describes that it is stationary in pattern. Further, ADF test is conducted on the resultant series **(Figure 5)** at α=0.05 and verify that the cement demand is stationary in nature as the statistical value h=1 and p-value=0.001. **Figure 6** and **Figure 7** represents the ACF and PACF plot for the resultant demand series. In **Figure 6**, significant spikes can be observed at lag 1 and lag 12, hence q=1 and Q =1. Likewise, in **Figure 7** significant spikes can be seen at lag 1 and lag 3 hence p =2 and P = 0. Based on the identified parameters the ARIMA model for the cement demand is ARIMA (2, 1, 1) $(0, 1, 1)_{12}$.

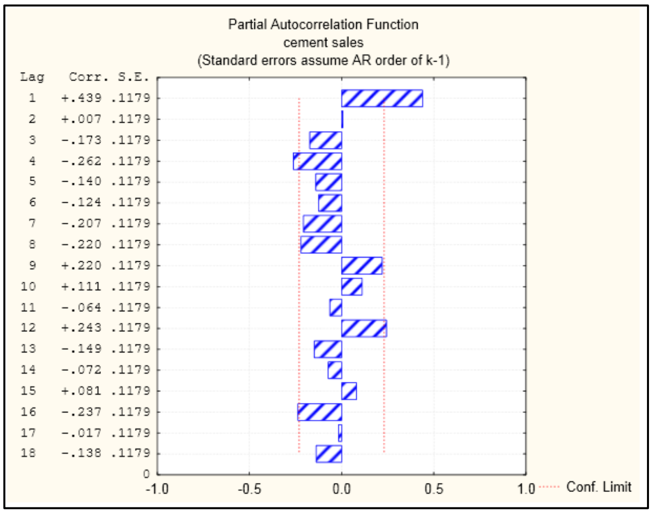

Figure 4 PACF plot of cement deman

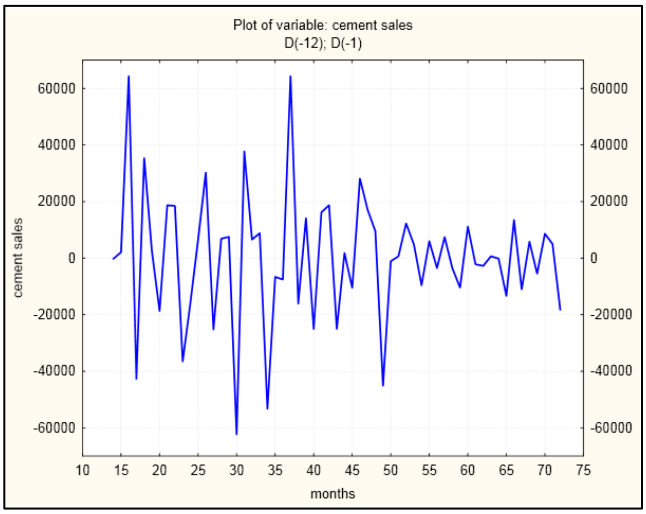

Figure 5 Time series plot for differenced cement demand

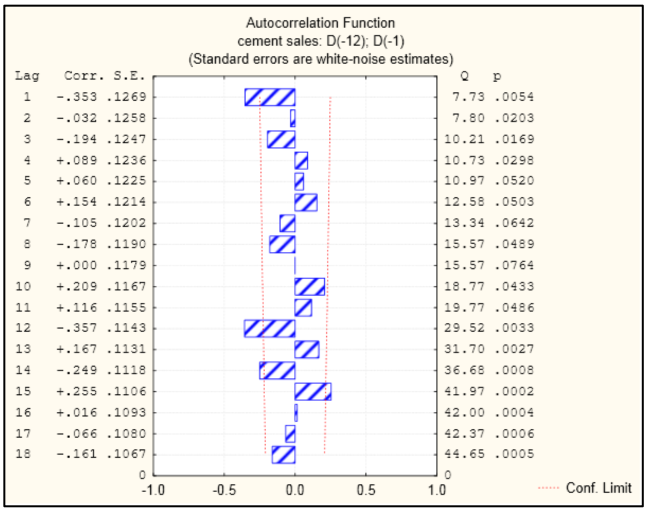

Figure 6 ACF plot for differenced cement demand

To identify the GARCH model for cement demand series, first the existence of the correlation and influence of ARCH effect is tested. To test the existence heteroscedasticity, the Ljung-Box-Pierce Q-test and the ARCH test is conducted on the return series **(Figure 5).** In Ljung-Box-Pierce Q-test, h=0 indicates that there is no significant correlation exist while h=1 specifies the presence of correlation. Correspondingly, in case of Engle ARCH test, h=1 indicates the existence of ARCH effect and h=0 indicates there is no ARCH effect. The summary of the Ljung-Box-Pierce Q-test for return and squared return is described in **Table 1** and **Table 2** respectively. The summary concludes that there is a significant correlation exist in raw returned and squared return of the demand when tested for up to 10, 15 and 20 lags of the ACF at $\alpha = 0.05$. Summary of ARCH test **(Table 3)** concluded that there is ARCH effect exist in the cement demand since h=1 and p-value < 0.05 tested at $\alpha = 0.05$ for lag 10, 15 and 20.

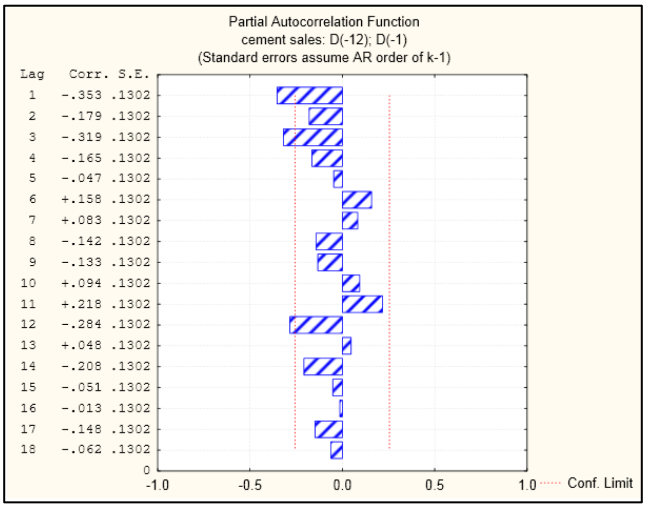

Figure 7 PACF plot for differenced cement demand

Table 1 Summary of Ljung-Box-Pierce Q-Test for return demand series

Lag	H	p-Value	Stat	Critical Value
10	1	0.0178	21.5178	18.307
15	1	0.0456	23.4199	24.9958
20	1	0.0248	34.2014	31.4104

Table 2 Summary of Ljung-Box-Pierce Q-Test for squared return of demand series

Lag	H	p-Value	Stat	Critical Value
10	1	0.0006	31.132	18.307
15	1	0	67.3172	24.9958
20	1	0	71.5217	31.4104

Table 3 Summary ARCH test for return demand series

Lag	H	p-Value	Stat	Critical Value
10	1	0.0093	23.411	18.307
15	1	0.0227	27.8242	24.9958
20	1	0.0397	32.3486	31.4104

The Ljung-Box-Pierce Q-test and ARCH test proved that cement demand data is heteroscedastic in nature. Hence, accurate calculation of safety stock level is crucial to manage the demand and inventory. Therefore, cement manufacturing company has taken as a case to study and analyze the performance of the ARIMA and ARIMA-GARCH model in terms of the parameters BWE and net-stock amplification. Small order of p and q is generally applicable to choose the best suitable model. The simple GARCH model able to capture most variability in the demand series. Normally, the GARCH (1, 1), GARCH (1, 2) or GARCH (2, 1) models are appropriate for modeling volatilities (Bollerslev *et al*., 1992). However, GARCH (0, 1), GARCH (0, 2) and GARCH (2, 2) models are included to identify the suitable models for the time varying demand as shown in **Table 4**. The idea behind this is to have a parsimonious model that can capture demand pattern as possible. The statistical tests, Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC) are done to identify the suitable model from the competing models. Those models have lesser value of AIC and BIC are treated as favorable models. The **Table 4**, described AIC and BIC value for six different models. Among the listed models GARCH (2, 1) has lesser AIC and BIC value therefore, taken as suitable model to forecast the demand variance.

Table 4 Different GARCH models with AIC and BIC value

Models	AIC	BIC
GARCH(0,1)	1502.9	1509.7
GARCH(1,1)	1382.1	1391.2
GARCH(0,2)	1394.2	1403.3
GARCH(1,2)	1384.1	1395.5
GARCH(2,1)	1380.1	1391.1
GARCH(2,2)	1382.1	1395.7

Different statistical test has been performed and parameters like coefficient and t-statistics are determined to evaluate the best-fitted model. The **Table 5** summarizes the estimated parameters for the model GARCH (1, 1). From the table it can be concluded that GARCH (1, 1) is satisfying the necessary conditions of a GARCH model i.e. $\alpha_1 + \beta_1 < 1$,$\beta_1 > \alpha_1$ $\alpha_1 > 0$ and $\beta_1 > 0$ and hence it is selected an appropriate model. Similarly, the estimated parameters for the model GARCH (1, 2) is defined in **Table 6**. The parameters justified that GARCH (1, 2) model is also satisfying the necessary conditions i.e. $\beta_j > \alpha_i$, $\alpha_i + \beta_j < 1$, $\alpha_i \geq 0$ and $\beta_j \geq 0$ where (i=1,2,…,q and j=1,2,…,p). The summary of the estimated parameters for the model GARCH (2, 1) are defined in **Table 7**. From the parameters, it can be said that GARCH (2, 1) is a suitable model.

Table 5 Summary of estimated parameters of the model GARCH (1, 1)

Parameters	Value	Standard error	t-Statistic
C	-312.05	4022.2	-0.0776
K	1.45E+08	0.00885	1.64E+10
GARCH(1)	0.5649	0.1304	4.3312
ARCH(1)	0.3543	0.1917	1.8478

Table 6 Summary of estimated parameters of the model GARCH (1, 2)

Parameters	Value	Standard error	t-Statistic
C	-312.05	4013.6	-0.0777
K	1.45E+08	0.0158	9.19E+09
GARCH(1)	0.5527	0.2571	2.15
ARCH(1)	0.3529	0.1933	1.8256
ARCH(2)	0.0172	0.3019	0.057

Table 7 Summary of estimated parameters of the model GARCH (2, 1)

Parameters	Value	Standard error	t-Statistic
C	-312.05	3489.5	-0.0894
K	1.45E+08	0.0065	2.25E+10
GARCH(1)	0	0.2352	0
GARCH(2)	0.4763	0.2550	1.8677
ARCH(1)	0.3892	0.2018	1.9285

Similarly, the parameters are estimated for GARCH (2, 2), GARCH (0, 1) and GARCH (0, 2) as described in **Table 8**, **Table 9** and **Table 10** respectively. From the tables, it can be observed that except GARCH (0, 2) rest other are satisfying the necessary conditions of a GARCH model and these are selected as suitable models. The model GARCH (0, 2) is violating the rule $\alpha_i + \beta_j < 1$ i.e. 0.42065+0.57935 > 0 (from **Table 10**). However, from **Table 4** it has been determined that GARCH (2, 1) has comparatively less AIC and BIC value as compared to rest of the GARCH models. Hence, GARCH (2, 1) is identified as the best-fitted model for prediction of the cement demand variance.

Table 8 Summary of estimated parameters of the model GARCH (2, 2)

Parameters	Value	Standard error	t-Statistic
C	-312.05	3570.3	-0.0874
K	1.45E+08	0.0153	9.5E+09
GARCH(1)	0	0.4078	0
GARCH(2)	0.4762	0.2646	1.8
ARCH(1)	0.3893	0.2032	1.9155
ARCH(2)	0	0.3156	0

Table 9 Summary of estimated parameters of the model GARCH (0, 1)

Parameters	Value	Standard error	t-Statistic
C	-312.05	725.13	-0.4303
K	1.45E+08	4.97E+06	29.2372
ARCH(1)	0.84997	0.25201	3.3727

Table 10 Summary of estimated parameters of the model GARCH (0, 2)

Parameters	Value	Standard error	t-Statistic
C	-312.05	1848.8	-0.1688
K	1.45E+08	0.04205	3.46E+09
ARCH(1)	0.4207	0.1280	3.2858
ARCH(2)	0.5794	0.1423	4.071

The goodness of fit of the ARCH or GARCH model is generally defined based on the residuals property and is more specifically on the standardized residuals (Takle, 2003). According to the assumptions considered in GARCH model, the residual follows an additive white Gaussian noise process i.e. model fit the data well and the residuals should be randomly independent and identically distributed to follow the normal distribution. If these assumptions are satisfied, then a model can be treated as a best-fitted model. **Figure 8** describes the relationship between residual (innovation), conditional standardized innovation and the return series of the identified fitted model i.e. GARCH (2, 1). In this figure, a volatility clustering can be observed in return series and innovation series. **Figure 9** summarizes the pattern for standardized innovation. Standardized innovation is the ratio of innovation and conditional standard deviation value. From the figure, it can be concluded that standardized innovation series is stable with slight clustering in pattern.

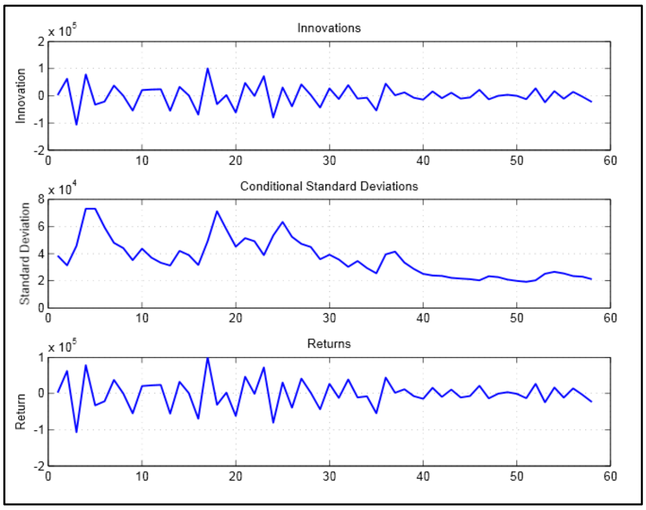

Figure 8 Relationship of return, estimated volatility and innovation series

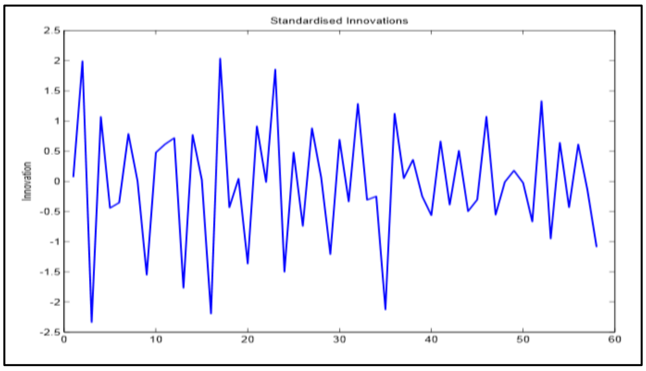

Figure 9 Time series plot for residuals of GARCH (2, 1)

To check the normality in the residual series a normal probability plots is plotted for the residuals from GARCH (2, 1) as presented in **Figure 10**. Most of the points are falling along the dashed straight line and representing that the residuals are following normal distribution (**Figure 10**). A model is said to be a successful model when there is no autocorrelation left in the standardized residual and squared standardized residual. The **Figure 11**, represent the ACF plot for the standardized residuals. All the spikes are within the boundary hence there is no correlation left. Further, to verify the existence of correlation and heteroscedasticity in residual series the Ljung-Box-Pierce Q-test and the ARCH test done for the standardized innovation series. The respective estimated values are defined in **Table 11** and **Table 12**. The observed values, h=0 and p-Value >= 0.05 represents it fail to reject the null hypothesis hence proved that there is neither correlation nor ARCH effect has left in the standardized innovation series. Therefore, GARCH (2, 1) selected as the best-fitted model and using this model 12-month ahead demand variance is predicted.

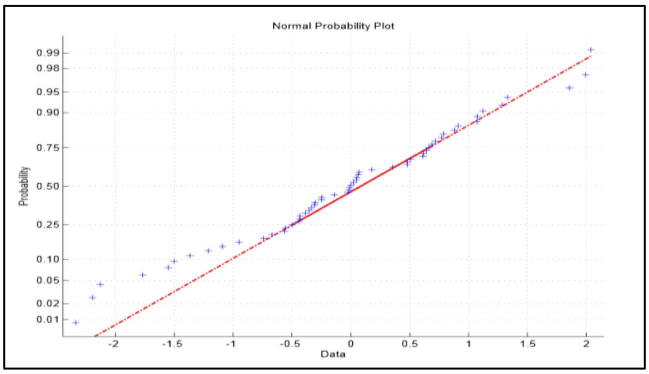

Figure 10 Normal probability plot for the residual series of model GARCH (2, 1)

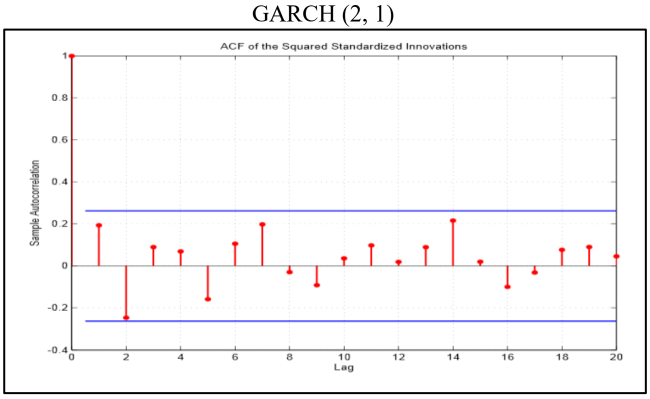

Figure 11 ACF of the squared standardized residuals of the model GARCH (2, 1)

Table 11 Summary of Ljung-Box-Pierce Q-test for standardized innovations series

Lag	H	p-Value	Stat	Critical value
10	0	0.2403	12.7111	18.307
15	0	0.2733	17.7998	24.9958
20	0	0.4491	20.1415	31.4104

Table 12 Summary of ARCH test for standardized innovations

Lag	H	p-Value	Stat	Critical value
10	0	0.2225	13.0206	18.307
15	0	0.258	18.0898	24.9958
20	0	0.4497	20.1317	31.4104

6. RESULTS AND DISCUSSION

The performance of the ARIMA and ARIMA-GARCH is measured through BWE and net-stock amplification ratio.

The order quantities (Q_t) are estimated using simple form of order up-to level i.e. ((R, S)) inventory control policy i.e. base-stock inventory policy using Equation (7) - (8) considering lead time (L) and review period (R) equals to one time period (Zhang, 2004; Luong & Phien 2007).

$$Q_t = \left(\hat{D}_t^L + z\hat{\sigma}_t^L\right) - \left(\hat{D}_{t-1}^L + z\hat{\sigma}_{t-1}^L\right) + D_{t-1} \quad (7)$$

$$Q_t = \left(\hat{D}_t^L - \hat{D}_{t-1}^L\right) + D_{t-1} \quad (8)$$

where, D_{t-1} represents the actual cement demand at t-1 period, term z denotes the service level to fulfill the order and it is assumed as 95% hence resulting z-value is1.96. The parameter $\hat{\sigma}_t^L$ is the forecasted demand variation during the lead-time while $\hat{\sigma}_{t-1}^L$ is the forecasted demand variation during the just previous period t-1. $\hat{D}_t^L$ and $\hat{D}_{t-1}^L$ represent the predicted mean demand during lead time at time period t and t-1. In the case of heteroskedastic demand series Equation 7 can be used to estimate the order quantity. Whereas, Equation (8) is applied for homoscedastic demand series where there is no change in demand variance. Using identified GARCH (2, 1) model 12-months ahead cement demand variance is forecasted and the mean demand are forecasted using identified model ARIMA (2, 1, 1) (0, 1, 1)12. Using forecasted mean and variance of demand, the safety-stock volumes ($z\hat{\sigma}_t^L / z\hat{\sigma}_{t-1}^L$) are updated in each replenishment period to calculate the order quantities using Equation (7). When the demand variance is not considered, the order quantities are estimated using Equation (8) i.e. without considering predicted demand variance using the GARCH model. Next, the BWE and net-stock amplification is calculated using Equations (9), Equation (10) respectively, and summaries in **Table 13**.

$$Bullwhip\ effect = \frac{order\ variance}{demand\ variance} \quad (9)$$

$$Net - stock\ amplification = \frac{varaince\ of\ net{-}stcok}{variance\ of\ demand} \quad (10)$$

Table 13 Estimated BWE and net-stock amplification

Model	BWE	Net-stock amplification
ARIMA	0.881	1.041
ARIMA-GARCH	0.953	1.024

As from the **Table 13**, it can be concluded that BWE approaches to one in the case of ARIMA-GARCH model. This signifies that BWE is less i.e. order variance value approximately equals to the demand variance value. While the estimated BWE in the case of ARIMA model is less than one indicating the damping scenario, i.e. variation in the order is low compared to variation in demand (Boute and Lambrecht, 2009). The BWE value in the case of ARIMA is comparatively higher than ARIMA-GARCH. The calculated net-stock amplification value in case the of ARIMA-GARCH model is bit less as compared to ARIMA model signifies less fluctuation in on-hand stock level. From the above study, it has been proved that ARIMA-GARCH model can estimate the order quantity with relatively high degree of accuracy through updating safety stock using predicted demand variance as compare to ARIMA model.

7. CONCLUSION

In this work, a hybrid approach of ARIMA and GARCH (ARIMA-GARCH) has been proposed to deal with heteroskedastic demand series. To describe the proposed model and to analyze its performance a case study example i.e. demand from cement manufacturing plant has been considered. By following different statistical procedure, the ARIMA and GARCH model for the cement demand series has identified. 12-months ahead mean and variance of demand are forecasted using identified ARIMA (2, 1, 1) (0, 1, 1)12 and GARCH (2, 1) model respectively. Then a comparative analysis has been carried out between ARIMA and ARIMA-GARCH to judge the performance through estimating BWE and net-stock amplification ratio considering base-stock policy. From the analysis, it is found that BWE and net-stock amplification values are comparatives less when the orders are estimated considering the forecasted time varying demand variance than without considering time varying demand variance. Hence, proved that the proposed model help in predicting the changing demand variance to calculate safety stock quantity in each replenishment period for accurate calculation of order volume. The proposed model can be applied by an organization to predict the demand variation to set safety-stock level and accurate calculation of order quantity. This further, help the organization to reduce the holding cost, customer service level and managing demand and order. The proposed model is verified for base-stock inventory control policy. Hence, the proposed model can be verified for other variant of the order-up-to level inventory control policy.

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Dr. Sanjita Jaipuria is presently working as Assistant Professor in the area of Operations and Quantitative Techniques in Indian Institute of Management, Shillong, India. Her teaching and research areas include forecasting, modelling and simulation, quality management and operations management. She published her research work in various international journals and presented in conferences.

Dr. S.S. Mahapatra is presently working as a Professor at the Department of Mechanical Engineering in National Institute of Technology, Rourkela, India. His areas of interest include multi-criteria decision-making, quality engineering, simulation, lean systems and service quality management. He has published 350 research papers in various international and national peer reviewed journals. He is an outstanding reviewer of many international journals. In addition, he is a visiting faculty to engineering schools in India and abroad.