OPERATIONS AND SUPPLY CHAIN MANAGEMENT
Vol. 14, No. 4, 2021, pp. 444 – 455
ISSN 1979-3561 | EISSN 2759-9363

Raw Material Order Allocation Problem Using Mixed Integer Linear Programming and Simulation

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ABSTRACT

This study aims to provide a conceptual model and to develop a mathematical model to determine order allocation in multi products, multi suppliers, multi carriers, and multi periods problem under uncertainty. The conceptual model describes the connection between related variables. The problem is formulated in mixed integer linear programming (MILP) model. MILP model objective function is to minimize supply chain costs which are purchasing cost, ordering cost, inventory holding cost, carrier cost, late delivery penalty cost, and low-quality penalty cost. In order to illustrate the applicability of the MILP model, a real-world case in cement industry is demonstrated. Based on historical data, the most common uncertainty factor is supplier delivery performance and product quality. Those factors are experimented in MILP model using Monte Carlo simulation. The integration between MILP model and Monte Carlo simulation shows that the proposed model resulted a global optimum solution.

Keywords: *order allocation, optimization, simulation, supplier selection, linear programming*

1. INTRODUCTION

The global industrial competition is getting more intense. Many players attempt to improve their business process in order to gain competitive advantage. They cooperate and coordinate amongst themselves to get more efficiency in their business operations. Higher efficiency helps a company achieve competitive advantage (Hosseini *et al.*, 2018). Many factors in today's global market have forced companies to gain a competitive advantage by focusing attention to their entire supply chain (Mendoza & Ventura, 2010). In a supply chain, sourcing is one of the most strategic aspect when a company attempts to reduce cost and improve competitiveness (Xia & Wu, 2007). One of the crucial steps in sourcing is supplier selection process. Selecting the best suppliers and allocating orders to the selected suppliers is a significant business process (Jolai *et al.*, 2011). Selecting the right suppliers is a key to the procurement process and represents a major opportunity for companies to reduce costs (Kumar *et al.,* 2018). A proper supplier selection process gives an opportunity to cost reduction that affects the company efficiency.

The aspect that becomes an issue in selecting supplier selection is the method for selecting the best supplier. The supplier selection problem can be either a single-sourcing problem, in which one supplier is selected to satisfy the firm's entire demand, or a multiple-sourcing problem, in which more than one supplier is selected (Hamdan & Cheaitou, 2017). Ustun & Demirtas (2008) in their paper said basically there are two kinds of supplier selection problem. In the first kind of supplier selection, one supplier can satisfy all the buyer's needs (single sourcing). The management needs to make only one decision: which supplier is the best. In the second type (multiple sourcing), no supplier can satisfy all the buyer's requirements. The use of single sourcing if a supplier can meet all the needs of buyer such as quality, quantity, and delivery, while multiple sourcing if there is no single supplier that can meet all the needs of buyers due to various limitations such as capacity, price, quality level, and delivery time (Ware *et al.,* 2014).

Many authors have published some research about multiple-sourcing selection. Those studies aimed to provide a procedure or a tool to select the best feasible alternative in decision making. Choudhary & Shankar (2013) developed mixed integer linear programming (MILP) model considering purchasing cost, order cost, inventory holding cost, late delivered product cost, using multi carriers for multi suppliers in multi periods. Wicaksono *et al.* (2018) also provided a MILP model considering discounts using a single carrier for multi products, multi suppliers in multi periods. Another research by Wicaksono *et al.* (2019), the authors integrated carrier selection into a DSSP model where multiple products are procured from multiple suppliers (Wicaksono *et al.*, 2019). Romero-Hernandez *et al.* (2021) proposed a modified Quality Function Deployment (QFD) approach to be an integrated framework for supply chain selection. This methodology is applied for a start up company to balance between fulfillment and availability of supply chain. A research conducted by Mujkic *et al.* (2018) shows that different supply chain mathematical models and formulations applied in sustainability dimensions. The research also describes that most authors adopted MILP as the modelling approach. However, only few research considering uncertainty and randomness in their paper. According to Chen *et al.* (2006), the supplier selection may involve several and different criteria, a combination of different decision models, group decision-making and various forms of uncertainty.

In this paper, we provide a conceptual model to show the multi products, multi suppliers, multi carriers, and multi periods problem in a diagram. We also develop a mathematical model in mixed integer linear programming (MILP) model to determine the optimum order allocation solution. We experiment with the model by generating random value in delivery performance and product quality as the uncertainty factors. These two factors are most common uncertainty factor and also have a financial impact, like a penalty cost to a supplier who cannot meet the buyer requirement. We use Monte Carlo simulation to generate random value and combine the result to the MILP model. The Monte Carlo method of statistical analysis uses random time series generated with spectral characteristics similar to the actual data time series used in producing the statistic fields (Stanford & Ziemke, 1994). Monte Carlo experimentation is the use of simulated random numbers to estimate some functions of a probability distribution (Gentle, 2010). Therefore, this study aims to provide a decision-making model to solve multi products, multi suppliers, multi carriers, and multi periods order allocation problem under uncertainty.

This paper is divided into 5 sections. Section 1 discusses about the background of this research. The next section presents a literature review and gap on previous research. Third section provides the development model, including conceptual model and MILP model. Section 4 shows data experiment in the real-world case. Finally, the last section discusses about conclusion and future research.

2. LITERATURE REVIEW

In supply chain management, supplier selection is a part of purchasing process. The stage of supplier selection starts from identifying, evaluating and selecting the best supplier in accordance with particular criteria. The purchasing function is increasingly seen as a strategic issue in organizations. Buyer and supplier relationships in manufacturing enterprises have received a great deal of attention (Chen, Lin & Huang, 2006). In traditional practices, first suppliers are selected, and then the buyer, by taking some other considerations and side constraints into account, makes the final decision on how much to order from each. But in the recent decade, researchers have concentrated on integrated approaches in which the issues of supplier selection and order allocation are simultaneously investigated (Jolai *et al.*, 2011).

Most of supplier selection and order allocation problem are multi objective issue. It means there is over one objective but conflicting each other. Pan & Wang (2014) presented a multi-objective model of order allocation using mixed integer linear programming. Mendoza & Ventura (2010) investigated supplier selection and order quantity allocation problem using non-linear programming and Power of Two (POT). The previous research mostly discussed quantitative aspect. Kumar *et al.* (2018) added qualitative aspect in their research. They developed supplier selection and order allocation model using Analytical Hierarchy Process (AHP) and Linear Physical Programming. Hamdan & Cheatiou (2017) added green criteria on their approach. Using fuzzy TOPSIS, AHP, optimization, they proposed the model to solve supplier selection and order allocation problem. Wicaksono *et al.* (2018) developed a mixed integer linear programming (MILP) model considering discount in multi products, multi suppliers and multi periods. Choudhary & Shankar (2013) developed MILP model to minimize purchasing cost, order cost, inventory holding cost, late delivered product cost using multi carriers for multi suppliers in multi periods but in a single product.

A list of literature review is depicted in **Table 1**. Many of these previous research already offered a multi-objective problems solution. The authors developed a model, but in limited variables. In this paper, we fulfill the drawback by examining all related variables into a single mathematical model. Only few literatures considered uncertainty and randomness into their model. Some of them are Hamdan & Cheatiou (2017), Jolai *et al.*, (2011), and Chen *et al.* (2006). The rest of the literature assume that all of the variables are constant and deterministic. We integrated the model with Monte Carlo simulation to accommodate uncertainty. Data experiment in real-world case is also applied to test the model. This study clearly aims to fulfill the gap in previous papers.

This research as mentioned above at least has three main contributions. The proposed model is consisted of more related variables. It makes the model more complex and closer to the real problem. The second one, integration Monte Carlo simulation into MILP model. This integration makes better approach than the previous research. The last contribution is about data experiment in real-world case. It is applied to test the feasibility of the proposed model.

3. PROPOSED MODEL

In this section, we provide a conceptual model and develop a mathematical model formulation. The conceptual model shows the relation between variable of products, suppliers, carrier, and periods. Mathematical model formulation shows indices, parameters, objective function, and decision variables of the order allocation problem. The problem for proposed model can be briefly described as follows: amount of multi products orders supplied by multi suppliers using multi carriers in multi periods. The objective function of this model is to minimize supply chain costs which are purchasing cost, order cost, inventory holding cost, carrier cost, late delivery penalty cost, and low-quality penalty cost.

Table 1 Gap research

No	Title (Year)	Tool	Variables															
			Cost*	Multi**	Discount	Rejection	Late Delivery	Backordering	Green Criteria	Shortage	Budget	Minimum Order	Operating Efficiency	Tech Level Satisfaction	Service Quality	Disruption Risk	Simulation	Uncertainty
1	Mixed integer linear programming model for dynamic supplier selection problem considering discounts (2018)	MILP	F	H	F	F	F	-	-	-	-	-	-	-	-	-	-	-
2	Joint decision of procurement lot-size, supplier selection, and carrier selection (2013).	MILP	F	H	F	F	F	-	-	-	-	-	-	-	-	-	-	-
3	Supplier selection and order allocation with green criteria: An MCDM and multi-objective optimization approach (2016).	Fuzzy Topsis, AHP, Bi Objective Integer & Non-Integer Programming	F	H	-	-	-	-	-	F	F	-	-	-	-	-	-	F
4	Integrating fuzzy TOPSIS and multi-period goal programming for purchasing multiple products from multiple suppliers (2011).	Fuzzy Topsis, Fuzzy AHP, Goal Programming	F	H	-	F	-	-	-	-	F	F	-	-	-	-	-	F
5	Supplier Selection and Order Allocation in Supply Chain (2018).	AHP, Linear Physical Programming	-	-	-	-	-	-	-	-	-	-	F	F	F	-	-	-
6	A serial inventory system with supplier selection and order quantity allocation (2010).	Non-Linear Programming, POT	F	H	-	F	-	-	-	-	-	-	-	-	-	-	-	-
7	Lot sizing and supplier selection with multiple items, multiple periods, quantity discounts, and backordering (2018).	MIP, Heuristics	F	H	F	-	-	F	-	F	-	-	-	-	-	-	-	-
8	A Multi-objective Model of Order Allocation under Considering Disruption Risk and Scenario Analysis in a Supply Chain Environment an Integrated Multi-objective Model for Order Allocation (2014).	Multi Objective MILP	F	H	-	F	-	-	-	-	-	-	-	-	-	F	-	-

Table 1 Gap research (con't)

No	Title (Year)	Tool	Variables															
			Cost*	Multi**	Discount	Rejection	Late Delivery	Backordering	Green Criteria	Shortage	Budget	Minimum Order	Operating Efficiency	Tech Level Satisfaction	Service Quality	Disruption Risk	Simulation	Uncertainty
9	A fuzzy approach for supplier evaluation and selection in supply chain management (2006)	Fuzzy Topsis	-	H	-	-	-	-	-	-	-	-	-	-	-	-	-	F
10	A mixed integer linear programming model for dynamic supplier and carrier selection problems (2019).	MILP	F	H	F	F	F	-	-	-	-	-	-	-	-	-	-	-
11	Supplier selection with multiple criteria in volume discount environments (2007).	AHP, Mixed Integer Programming	H	-	F	F	F	-	-	-	-	-	-	-	-	-	-	-
12	An integrated multi-objective decision-making process for multi-period lot-sizing with supplier selection (2008).	ANP, Multi Objective MILP	F	H	-	F	-	-	-	-	-	-	-	-	-	-	-	-
13	A mixed-integer non-linear program to model dynamic supplier selection problem (2014).	Mixed Integer Non-Linear Programming	H	H	-	F	F	-	-	-	-	-	-	-	-	-	-	-
14	A Heuristic Approach for Determining Lot Sized and Schedules Using Power-of-Two Policy (2007)	POT	H	H	-	-	-	-	-	-	-	-	-	-	-	-	-	-
15	This paper research (2020)	MILP, Monte Carlo simulation	F	F	-	F	F	-	-	-	-	-	-	-	-	-	F	F

Note:
* : Purchasing cost, order cost, inventory holding cost, carrier cost.
** : Multi products, multi suppliers, multi carriers, multi periods.
F = Fully available; H = Not fully available; - = Not available

3.1 Conceptual Model

A conceptual model is a diagram that shows the connection between variables. The model represents the real problem as a system. In this paper, the variables are suppliers, carrier modes, raw materials as treated as products, and periods. **Figure 1** shows the conceptual model.

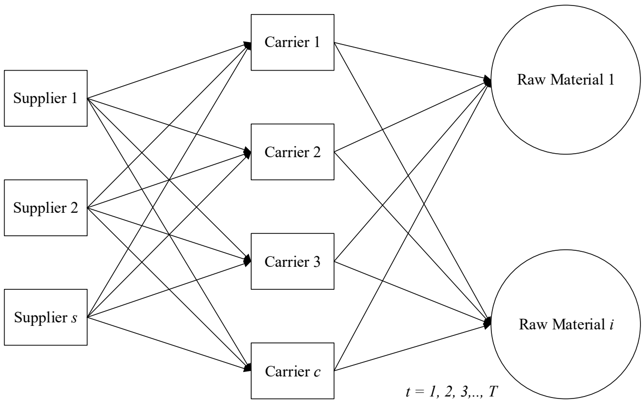

Figure 1 Conceptual model

The direction of the arrow to the right shows that the flow of material from the origin point (supply side) to fulfill the destination point (demand side). Each arrow brings some values. The value is the amount of order that must be delivered to fulfill the raw material demands and to satisfy the constraints. Variables also have some limitation called constraint. Variable of supplier has a supplier capacity constraint, variable of carrier has a carrier capacity constraint, and variable of raw material has a storage capacity constraint. The conceptual model describes how the demand of raw material i is fulfilled by supplier s that it used carrier c in period t.

3.2 Model Assumption

These models are restricted by some assumptions. Demands are known and constant. The rest of order due to late delivery are allowed to be delivered in the next period. Product with low quality is accepted and stored at the buyer storage, but penalty cost is still charged. Shortages are not allowed. Carrier costs apply to supplier, not buyer. Carrier costs are charged at the full truckload (FTL) capacity tariff despite the amount which is carried less than FTL capacity. Lead time is constant. In this model, raw materials and products are substitutes.

3.3 Model Indices, Parameters, and Decision Variables

These followings are model indices, parameters, and decision variables for mathematical formulation in the next sub section.

Indices

i	raw material; 1,2,…,I
s	supplier; 1,2,…,S
c	carrier; 1,2,…,C
t	period; 1,2,…,T

Parameters

$\boldsymbol{UC_{is}}$	unit price of raw material i supplied by supplier s
$\boldsymbol{D_{it}}$	demand of raw material i in period t
$\boldsymbol{O_{ist}}$	order cost of raw material i for supplier s in period t
$\boldsymbol{B_{isct}}$	carrier cost c is used by supplier s to supply raw material i in period t
$\boldsymbol{L_{ist}}$	raw material i late delivery rate supplied by s in period t
$\boldsymbol{Pl_{ist}}$	raw material i late delivery penalty cost supplied by supplier s in period t
$\boldsymbol{Q_{ist}}$	raw material i low quality rate supplied by supplier s in period t
$\boldsymbol{Pq_{ist}}$	raw material i low quality penalty cost i supplied by supplier s in period t
$\boldsymbol{HC_{it}}$	holding cost of raw material i in period t
$\boldsymbol{IA_{i}}$	inventory level of raw material i in initial period (t = 0)
$\boldsymbol{CC_{isct}}$	carrier c capacity to supply raw material i used by supplier s in period t
$\boldsymbol{CS_{ist}}$	supplier s capacity s to supply raw material i in period t
$\boldsymbol{SC_{it}}$	storage capacity of raw material i in period t
$\boldsymbol{n_{ist}}$	minimum order level of raw material i supplied by supplier s in period t according to management policy
$\boldsymbol{MO_{ist}}$	minimum order level of raw material i supplied by supplier s in period t according to supplier requirement
$\boldsymbol{M}$	a big number

Decision Variables

$\boldsymbol{X_{isct}}$	amount of raw material i order supplied by supplier s using carrier c in period t
$\boldsymbol{N_{isct}}$	number of carrier c used by supplier s to supply raw material i in period t
$\boldsymbol{INV_{it}}$	inventory level of raw material i in period t
$\boldsymbol{V_{st}}$	binary variable. 1 if supplier s is selected, 0 if otherwise

3.4 Model Formulation

According to model indices, parameters, and decision variables above, a mathematical formulation may be as stated as follows:

$$Minimize\ Z = Z_1 + Z_2 + Z_3 + Z_4 + Z_5 + Z_6 \tag{1}$$

$$Z_1 = \sum_i \sum_s \sum_c \sum_t X_{isct} * UC_{is} \tag{1a}$$

$$Z_2 = \sum_i \sum_s \sum_t O_{ist} * V_{st} \tag{1b}$$

$$Z_3 = \sum_i \sum_t INV_{it} * HC_{it} \tag{1c}$$

$$Z_4 = \sum_i \sum_s \sum_c \sum_t N_{isct} * B_{isct} \tag{1d}$$

$$Z_5 = \sum_i \sum_s \sum_c \sum_t X_{isct} * L_{ist} * Pl_{ist} \tag{1e}$$

$$Z_6 = \sum_i \sum_s \sum_c \sum_t X_{isct} * Q_{ist} * Pq_{ist} \tag{1f}$$

Subject to

$$\sum_s \sum_c X_{isct} + IA_{it} \geq D_{it} \qquad \forall i = 1, \dots, I;\ \forall t = 1 \tag{2a}$$

$$\sum_s \sum_c X_{isct} + INV_{i(t-1)} \geq D_{it} \qquad \forall i = 1, \dots, I;\ \forall t \neq 1 \tag{2b}$$

$$\sum_s \sum_c X_{isct} - \sum_s \sum_c X_{isct} * L_{isct} + IA_i = D_{it} + INV_{it} \qquad \forall i = 1, \dots, I\ ;\ \forall t = 1 \tag{3a}$$

$$\sum_s \sum_c X_{isct} - \sum_s \sum_c X_{isct} * L_{isct} + \sum_s \sum_c X_{isc(t-1)} * L_{isc(t-1)} + INV_{i(t-1)} = D_{it} + INV_{it} \qquad \forall i = 1, \dots, I\ ;\ \forall t \neq 1 \tag{3b}$$

$$X_{isct} \leq N_{isct} * CC_{isct} \qquad \forall i = 1, \dots, I;\ \forall s = 1, \dots, S;\ \forall c = 1, \dots, C;\ \forall t = 1, \dots, T$$

$$\sum_c X_{isct} \leq CS_{ist} \qquad \forall i = 1, \dots, I;\ \forall s = 1, \dots, S;\ \forall t = 1, \dots, T$$

$$\sum_i \sum_c X_{isct} \leq M * V_{st} \qquad \forall s = 1, \dots, I;\ \forall t = 1, \dots, T \tag{6}$$

$$INV_{it} \leq SC_{it} \qquad \forall i = 1, \dots, I;\ \forall t = 1, \dots, T \tag{7}$$

$$\sum_c X_{isct} \geq n_{ist} * D_{it} \qquad \forall i = 1, \dots, I;\ \forall s = 1, \dots, S;\ \forall t = 1, \dots, T \tag{8}$$

$$\sum_c X_{isct} \geq MO_{ist} \qquad \forall i = 1, \dots, I;\ \forall s = 1, \dots, S;\ \forall t = 1, \dots, T \tag{9}$$

$X_{isct}, N_{isct} \geq 0$ and integer

$$\forall i = 1, \dots, I;\ \forall s = 1, \dots, S;\ \forall c = 1, \dots, C;\ \forall t = 1, \dots, T$$

$$V_{st} \in \{0,1\} \qquad \forall s = 1, \dots, I;\ \forall t = 1, \dots, T$$

The objective function (1) is to minimize purchasing cost (1a), order cost (1b), inventory holding cost (1c), carrier cost (1d), late delivery penalty cost (1e), and low-quality penalty cost (1f). In this proposed model, there are several constraints. The demand fulfillment constraint (2a and 2b) is to ensure demand each period is fulfilled. Inventory balance constraint (3a and 3b) states balance of inventory level in period *t*. This constraint also to ensure the demand in period *t* is fulfilled by inventory level in period *t* and the amount of late order in period *t-1*. Carrier capacity constraint (4) is to ensure the amount of order not exceed the capacity of carrier. This constraint also decides number of carriers needed. Supplier capacity constraint (5) states the amount of order is not to be allowed to exceed supply capacity of supplier. Ordering cost constraint (6) is to ensure order cost applied if the supplier is selected. Storage capacity constraint (7) is to ensure the amount of order not exceed buyer storage capacity. Management policy constraint (8) is to ensure every supplier receives minimum order according to management company policy. Supplier requirement constraint (9) is to ensure minimum order must be greater equal than supplier requirements. Constraint number 10 forces integer and non-negativity value. Finally, constraint number 11 is a binary variable constraint.

4. EXPERIMENTAL DATA

In section 4, we discuss how the proposed model demonstrated through data experiment. Data from real-world case in a cement industry is collected. It aims for illustrating how the model gain the solution. In cement production, a cement company needs some types of raw materials. They are limestone, clay, iron sand, silica sand, trass, and gypsum. Limestone and clay are usually mined from the company quarry itself. In this selected cement company, the gypsum must be purchased from single dedicated supplier. The rest of raw materials which are iron sand, silica sand, and trass, must be purchased from specific different suppliers. These suppliers are multiple dedicated suppliers. It means, based on management company policy, a whole aggregate order must be split into all suppliers. The result of field observation, iron sand is supplied by supplier D and E, trass is fulfilled by supplier F, G, and H, then silica sand is purchased from supplier M, N, and O. **Figure 2** depicts which suppliers fulfill what raw materials.

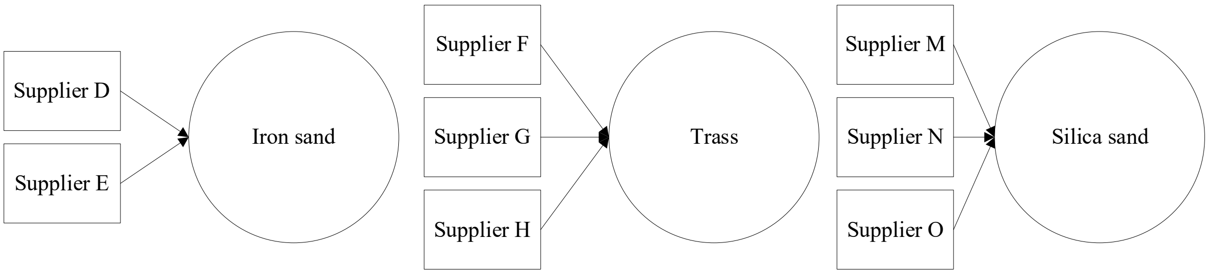

Figure 2 Raw material supplier

Data collection is started from unit price of raw materials to minimum order level as stated as in the previous section. We put the data into the parameter of the proposed model. All gathered data is assumed has no change along time horizon. A whole data parameter can be referred at Appendix.

We experiment parameter of raw material late delivery rate (List) and raw material low quality rate (Qist) as uncertainty factors before running the full mathematical model into optimization software called Lingo. Monte Carlo simulation is applied to imitate and to simulate the pattern of supplier delivery performance and product quality based on historical data. The result of the simulation effects on the amount of penalty cost suppliers must pay to the company. We divide the late delivery penalty cost into three events. It depends on the amount of supplier delivery that is sent on time according to the company order. The first event is no penalty applied, which means all the raw material completely delivered on time. The second event is if the supplier could only deliver 50% to 99% of raw materials ordered on the right time, they may be charged 2.5% penalty. Maximum penalty 5% is applied, if the suppliers send under 50% of raw materials ordered on schedule. We count the frequency of events according to the delivery performance from historical data. Then, we convert the frequency distributions to cumulative frequency/probability distributions. Put the interval of random numbers on each event. Tersine (1994) states sample at random from the cumulative probability distributions to determine specific variable values to use in the simulation. For instance, **Table 2** shows the result of supplier D late delivery rate distribution.

Table 2 Supplier D late delivery rate distribution

Penalty Event	Frequency	Cum. Freq	% Freq	% Cum. Freq	Random Numbers
No Penalty	22	22	63%	63%	01-63
Penalty 2.5%	2	24	6%	69%	64-69
Penalty 5%	11	35	31%	100%	70-100

We do over the prior step each raw material for all suppliers. The key result from example illustration in **Table 2** is the value of random numbers that will determine supplier delivery performance. Random numbers between 0 to 100 range are generated using spreadsheet for four periods. The result of generated random numbers will be matched to the random numbers in accordance with each supplier late delivery rate distribution. **Table 3** shows the result of Monte Carlo simulation delivery rate all suppliers for four periods. It will be an input in the mathematical formulation for parameter L_{ist}.

Table 3 Late delivery penalty monte carlo simulation

Period	1								2							
Supplier	D	E	F	G	H	M	N	O	D	E	F	G	H	M	N	O
Random Numbers	73	91	25	17	75	30	65	85	31	36	20	58	87	16	2	21
Penalty	2,5%	5%	0%	0%	2,5%	0%	3%	5%	0%	0%	0%	3%	5%	0%	0%	0%
Period	3								4							
Supplier	D	E	F	G	H	M	N	O	D	E	F	G	H	M	N	O
Random Numbers	41	53	82	82	82	79	96	82	70	78	60	44	23	13	23	69
Penalty	0%	0%	5%	3%	5%	3%	5%	3%	2,5%	5%	3%	0%	0%	0%	0%	3%

Another uncertainty factor as mentioned earlier is product/raw material quality rate. The company owns raw material quality standard as shown as **Table 4**. A raw material has two chemical parameters that must be met. If the suppliers deliver raw materials that do not meet the quality standard, they will be charged. The amount of penalty charge is measured from the quality range of product delivered by the supplier to the quality standard.

Table 4 Raw material quality standard

Raw Material	Parameter	Standard
Iron sand	Fe_2O_3	Min 50%
	H_2O	Max 5%
Trass	$SiO_2 + R_2O_3$	Min 75%
	H_2O	Max 10%
Silica sand	SiO_2	Min 90%
	H_2O	Max 6%

Historical data of raw material quality rate is collected from all suppliers. We analyze the data using statistics descriptive in a spreadsheet. The output is used to calculate the number of class interval (k) and the size of class interval (p) in Sturges Rules formula. According to Brkic (1991) the formula is

$$k = 1 + 3{,}3 \log n \tag{12}$$

$$p = \frac{R}{k} \tag{13}$$

Where n express the total number of observations the dataset and R is the data ranges.

The result of calculation will be used to set the lower bound and the upper bound in making the frequency distributions and cumulative frequency/probability distributions. The frequency distributions value is obtained from how often those value emerge in between the lower bound and the upper bound data observation ranges. Then, we put the interval of random numbers on every class interval. For example, **Table 5** shows the result of iron sand quality distribution for parameter Fe2O3 delivered by supplier D.

Table 5 Iron sand quality distribution for parameter Fe_2O_3

Lower Bound	Upper Bound	Median	Frequency	Cum. Freq	% Freq	% Cum. Freq	Random Numbers
33,89	39,61	36,75	3	3	2%	2%	01-02
39,62	45,34	42,48	3	6	2%	4%	03-04
45,35	51,07	48,21	6	12	4%	9%	05-08
51,08	56,80	53,94	8	20	6%	14%	09-13
56,81	62,53	59,67	73	93	52%	66%	14-65
62,54	68,26	65,40	32	125	23%	89%	66-88
68,27	73,99	71,13	8	133	6%	95%	89-94
74,00	80,12	77,06	7	140	5%	100%	95-100

Table 6 Low quality penalty monte carlo simulation

Supplier	D				E			
Period	1	2	3	4	1	2	3	4
Random Numbers (Fe_2O_3)	62	93	51	90	73	72	51	13
Quality Rate Simulation (Fe_2O_3)	59,67%	71,13%	59,67%	71,13%	59,25%	59,25%	59,25%	48,89%
Random Numbers (H_2O)	60	7	15	65	14	16	38	31
Quality Rate Simulation (H_2O)	5,66%	3,20%	4,02%	5,66%	3,89%	4,41%	4,93%	4,41%
Penalty (Fe_2O_3)	-	-	-	-	-	-	-	1,11%
Penalty (H_2O)	0,66%	-	-	0,66%	-	-	-	-
Low Quality Rate (Fe2O3 + H_2O)	0,66%	0,00%	0,00%	0,66%	0,00%	0,00%	0,00%	1,11%

We repeat the previous step each raw material in every parameter quality for all suppliers. The key point from **Table 5** is the value of random numbers in every class interval. Random numbers between 0 to 100 range are generated using spreadsheet for four periods. The result of generated random numbers will be matched to the random numbers under each supplier's raw material quality distributions. **Table 6** shows the result of the example Monte Carlo simulation in low quality rate for supplier D and supplier E. This result as an input in the mathematical formulation for parameter Q_{ist}.

As a multiple dedicated suppliers problem, we discuss the minimum percentage for order to the suppliers with the company management. This is to fulfill the data of management policy constraint (nist). The constraint is a distinctive constraint. It makes the order must be divided to all suppliers because of the suppliers have a high bargaining power to the company. So, the company decide to keep ordering yet as minimum as possible. We also ask to the suppliers to determine the minimum order, as stated as in supplier requirement constraint (constraint number 9), that they required. **Table 7** shows raw material minimum order each supplier according to the buyer management policy and supplier requirements.

Table 7 Raw material minimum order

Raw Material	Supplier	Management Policy (% of Demand)	Supplier Requirements (Ton)
Iron sand	D	40%	1500
	E	40%	3000
Trass	F	25%	3000
	G	25%	5000
	H	25%	3500
Silica sand	M	30%	3000
	N	30%	4000
	O	30%	3500

All completed data parameters are inputted into the model in the spreadsheet. We run the full model in Lingo 11.0.0.20 to satisfy raw materials demand shown in **Table 8**. The total demand for iron sand is 25700 ton, trass is 105.470 ton, and silica sand is 107810 ton. The demands are for four periods. The specification computer that we used is AMD Ryzen 7 2700U CPU Processor and 8 GB RAM.

Table 8 Raw materials demand (ton)

Raw Material	Period			
	1	2	3	4
Iron sand	5.900	3.500	8.150	8.150
Trass	19.970	23.250	30.000	32.250
Silica sand	26.110	15.470	36.110	30.120

The result of software optimization shows a global optimum solution has been found. The value of objective function on this case is IDR 57,8 billion. Z1 (purchasing cost) is IDR 29,1 billion, Z2 (order cost) is IDR 3,8 billion, Z3 (inventory holding cost) is IDR 0,15 billion, Z4 (carrier cost) is IDR 23,9 billion, Z5 (late delivery penalty cost) is IDR 0,52 billion, and Z6 (low-quality penalty cost) is IDR 0,27 billion. All developed constraints are fully satisfied. All multiple dedicated suppliers supply the raw materials.

The total amount of order for iron sand is 22900 ton, trass 106408 ton, and silica sand is 108894 ton. It is for four periods. As we know, the total amount of iron sand is lower than the demand as mentioned earlier. This is because the inventory level of iron sand in initial period (IAit) is greater than zero, which is 4.818 ton. Otherwise, since trass and silica have no initial inventory level (IAit), the total amount of order for trass and silica sand are lesser than the demand. **Table 9** and **Table 10** show the details of the decision variables, which the result from the optimization.

Table 9 Optimization result of decision variables (X_{isct} and N_{isct})

Raw Material	Supplier	Carrier	X_{isct}				N_{isct}			
			Period				Period			
			1	2	3	4	1	2	3	4
Iron sand	D	C1	20	0	20	20	1	0	1	1
		C2	2.340	1.500	3.240	3.240	78	50	108	108
	E	C1	0	0	20	20	0	0	1	1
		C2	3.000	3.000	3.240	3.240	100	100	108	108
Trass	F	C3	5.000	19.912	7.504	8.064	625	2489	938	1.008
		C4	0	0	0	0	0	0	0	0
		C5	0	0	0	0	0	0	0	0
	G	C3	0	0	0	0	0	0	0	0
		C4	10.096	5.816	7.512	8.064	1.262	727	939	1.008
		C5	0	0	0	0	0	0	0	0
	H	C3	0	0	0	0	0	0	0	0
		C4	5.000	5.816	7.504	16.120	625	727	938	2.015
		C5	0	0	0	0	0	0	0	0
Silica sand	M	C3	0	0	0	0	0	0	0	0
		C4	11.024	4.648	10.840	9.048	1.378	581	1.355	1.131
		C5	0	0	0	0	0	0	0	0
	N	C3	0	0	0	0	0	0	0	0
		C4	7.840	10.256	10.840	9.040	980	1.282	1.355	1.130
		C5	0	0	0	0	0	0	0	0
	O	C3	0	0	0	0	0	0	0	0
		C4	7.838	4.648	10.840	12.032	980	581	1.355	1.504
		C5	0	0	0	0	0	0	0	0

In **Table 9**, the optimization result of the decision variables is provided. The amount of raw material i order supplied by supplier s using carrier c in period t is notated by Xisct. In total, supplier D supplies 10.380 ton and supplier E supplies 12.250 ton of iron sand. To fulfill the demand of trass, the buyer order from supplier F of 40480 tons, supplier G of 31.488 ton, and supplier H of 34.440 ton. The silica sand order is divided to supplier M of 35560 tons, supplier N of 37.976 ton and supplier of O of 35.358 ton. In the next column of X*isct* in **Table 9**, there is also provided N*isct*. This decision variable shows the number of carriers needed. Supplier D needs 3 units of 20 ton truck capacity (C1) and 344 units of 30 ton truck capacity (C2) to supply iron sand in total. Supplier E, as also an iron sand supplier, requires 2 units of C1 and 416 units of C2 for four periods. Supplier F merely needs 5.060 units of 8 ton truck capacity (C3) to deliver all trass order. Supplier G and supplier H need less carrier than supplier F to fulfill the order. Supplier G needs 3.936 units of 8 ton truck capacity (C4) and supplier H requires 4.305 units. All suppliers of silica sand need C4, which is the truck with 8 ton capacity, to carry all their orders. Supplier M requires 4.445 units, supplier N needs 4.747 units, and supplier O should prepare 4.420 units to deliver the order. Details of Xisct and Nisct in every period can be traced in **Table 9**. The developed model also optimizes the inventory level of raw material i in period t. It is notated by INVit. This decision variable aims to determine the amount of inventory that must be stored to meet the demands. **Table 10** shows the optimization result of INVit decision variable.

Table 10 Optimization result of decision variable (INV_{it})

Raw Material	Period			
	1	2	3	4
Iron sand	4.069	5.278	3.648	1.774
Trass	1,2	7.984	2	737
Silica sand	4,1	4.674	0	784

The optimum inventory level in the last period for iron sand is 1774 ton, trass is 737 ton, and silica sand is 784 ton. Inventory level can not to be summed over periods, so we count the average of inventory level. This is because the inventory level in period t affects the number of orders for period t + 1. The result of inventory level average is 3.962 ton for iron sand, 2.181 ton for trass, and 1.365 ton for silica sand.

5. CONCLUSION

In today's business competition, a proper supply chain management practice offers the opportunity to reduce costs and improves the company's competitiveness. As a part of practices, supplier selection and order allocation process have important roles. A structured and comprehensive method may be needed to optimize the value. A conceptual model that provided in this paper attempts to build the order allocation in multi products, multi suppliers, multi carriers, and multi periods problem under uncertainty more structured and comprehensive. This study also develops a mathematical formulation properly in MILP model to optimize the supply chain costs as an objective function. Monte Carlo simulation applies to the model to overcome the uncertainty issue in supplier delivery performance and product quality. The proposed model that experimented in real-world case successfully provided a global optimum solution. The research contributes to provide a powerful guide for company decision makers in determining supplier selection and order allocation. Future research should add more relevant variables to provide a solution for more complex problems. A sensitivity analysis is necessary to be considered to find out the impact of changes in uncertainty variables on the result of the models. Furthermore, because of software capability, more complex problems will consume much time. A heuristic method should be proposed to overcome this issue.

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APPENDIX 1: COSTS (IDR), STORAGE CAPACITY (TON) AND INITIAL INVENTORY LEVEL (TON)

Raw Material	Unit Cost	Order Cost	Late Del Penalty Cost	Low Qual Penalty Cost	Holding Cost	Storage Capacity	Initial Inv
	IDR	IDR	IDR	IDR	%	Ton	Ton
Iron sand	425.000	118.520.800	425.000	425.000	2%	15000	4818
Trass	85.000	119.270.800	85.000	85.000	2%	40000	0
Silica sand	95.000	119.770.800	95.000	95.000	2%	45000	0

APPENDIX 2: CARRIER COSTS (IDR)

Raw Material	Supplier D		Supplier E		Supplier F			Supplier G, H, M, N, and O		
	C1	C2	C1	C2	C3	C4	C5	C3	C4	C5
Iron sand	10.500.000	12.000.000	10.500.000	12.000.000						
Trass					548.000	1.800.000	3500000	432.000	548.000	1.800.000
Silica sand								432.000	548.000	1.800.000

APPENDIX 3: CARRIER CAPACITY (TON)

Raw Material	Supplier D		Supplier E		Supplier F			Supplier G, H, M, N, and O		
	C1	C2	C1	C2	C3	C4	C5	C3	C4	C5
Iron sand	20	30	20	30						
Trass					8	15	30	5	8	15
Silica sand								5	8	15

APPENDIX 4: SUPPLIER CAPACITY AND MINIMUM ORDER (TON)

Raw Material	Supplier	Supplier Capacity	Minimum Order
		Ton	Ton
Iron sand	D	9000	1500
	E	9500	3000
Trass	F	35000	3000
	G	33000	5000
	H	34000	3500

Raw Material	Supplier	Supplier Capacity	Minimum Order
		Ton	Ton
Silica sand	M	40000	3000
	N	38000	4000
	O	39000	3500

APPENDIX 5: MONTE CARLO SIMULATION

		Late Delivery				Low Quality			
Raw Material	Supplier	1	2	3	4	1	2	3	4
Iron sand	D	2,50%	0,00%	0,00%	2,50%	0,66%	0,00%	0,00%	0,66%
	E	5,00%	0,00%	0,00%	5,00%	0,00%	0,00%	0,00%	1,11%
Trass	F	0,00%	0,00%	5,00%	2,50%	2,08%	1,00%	0,00%	2,37%
	G	0,00%	2,50%	2,50%	0,00%	0,00%	0,10%	0,10%	1,65%
	H	2,50%	5,00%	5,00%	0,00%	1,20%	0,00%	0,00%	0,00%
Silica sand	M	0,00%	0,00%	2,50%	0,00%	0,01%	1,35%	0,01%	3,17%
	N	2,50%	0,00%	5,00%	0,00%	1,35%	0,27%	3,18%	3,18%
	O	5,00%	0,00%	2,50%	2,50%	3,25%	1,08%	4,25%	0,28%

APPENDIX 6: OBJECTIVE FUNCTION (IN MILLION IDR)

Z1	Z2	Z3	Z4	Z5	Z6	Total
29.122	3.817	151	23.291	525	273	57.808

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