

Inventory Modelling for technology generation products under uncertain trade credit terms and imprecise procurement costs

Gaurav Nagpal

Birla Institute of Technology and Science Pilani, Rajasthan, India
Email: gaurav19821@gmail.com (*Corresponding Author*)

Udayan Chanda

Birla Institute of Technology and Science Pilani, Rajasthan, India
Email: udayanchanda@pilani.bits-pilani.ac.in

Alok Kumar

FORE School of Management, New Delhi, India
Email: alok@fsm.ac.in

Naga Vamsi Krishna Jasti

Birla Institute of Technology and Science Pilani, Rajasthan, India
Email: jasti1982@gmail.com

ABSTRACT

The inventory policies for any product under the trade credit mechanism are influenced by the procurement price per unit and the credit period offered by the seller to the buyer. This paper develops an inventory model for the technology generations under the imprecise trade credit period and the imprecise procurement cost. It considers the demand that is credit-linked and governed by innovation diffusion as well. The imprecise nature of the parameters is captured by the use of fuzzy numbers. The trapezoidal membership function has been used to fuzzify the profit function with the imprecise parameters, and then the centroid method is used to de-fuzzify the profit. The numerical illustrations have been performed, followed by the sensitivity analysis with the launch timing of the second generation product. A few important implications for the inventory practitioners and the possible extensions of this work have also been discussed.

Keywords: *inventory modeling, innovation diffusion, technology generations, trade credit, fuzzy logic, imprecise variables*

1. INTRODUCTION

If one observes the innovation products in the industry context, it can be discovered that these products vary from the conventional products by their time-varying demand pattern and shorter product life cycle (Goldman, 2007). Also, in the case of the innovation products, it has been often observed that multiple successive versions or so-called generations of the products co-exist in the marketplace, and cannibalize the demand of each other through the substitution effect (Speece and Macmachlan, 1995).

Technology diffusion is characterized by short product life cycles, and faster cannibalization by the newer product generations (Kreng and Jyun, 2013). The firms in

this business have to embrace cannibalization pro-actively with the changing customer preferences to prevent the competitors from eating their market share. A good example of such a firm is Apple Inc. which does not shy away from launching a new iPhone. Rather, it makes sure that the newer product is available on the shelf, knowing well that it will cannibalize the sales of the earlier models. Another example of such a firm is Google which launched Google Quick Scroll which allows faster search and jeopardized its Advertisement Platform. If Google would have shied away from doing this, Twinword Finder would have taken a part of its browser search tool business.

It is often found in the modern technology-driven world that whenever a new version of an innovation product gets launched, the demand for the earlier versions witnesses either a decline or atleast a fall in the growth rate. This happens because of the interplay between the markets of the multiple versions. If this phenomenon is not captured during the inventory planning of such products, it will lead to higher supply chain costs and lesser efficiencies. Nagpal *et al.* (2021) also emphasized the lack of inventory modeling literature on substitutable technology generations while doing a half-century review of the literature on modeling for substitutable products.

With the increasing competition in the industries today, the business functions can no longer afford to work in silos, since bringing efficiencies in the supply chain requires integrated processes and predictive cost-benefit analysis among others (Msimangira and Venkatraman, 2014). Businesses need to have inventory efficiency (Manikas and Patel, 2017). Moreover, this need for inventory efficiency is relevant for all three types of inventories (Manikas, 2017).

However, there are factors other than innovation characteristics also that influence the demand. Not only is the demand for innovation products governed by the

Theory of diffusion of innovations, but also, it is linked to the credit terms available in the marketplace. The role of the credit period in boosting the demand in modern trade cannot be undermined. . The higher credit terms to a customer help in boosting the demand, and therefore, are often used as a strategic tool for faster penetration of the product, or to have an edge over the value proposition being offered by the competing sellers in the market (Fabbri and Klapper, 2016).

The practical need to apply the fuzzy logic to this business context is very much relevant to be understood. Although the credit terms are generally pre-agreed upon between the buyer and seller in the modern trade, it would not be rare to observe cases where the actual credit terms are much different from the agreed-upon credit terms depending upon the working capital availability with the buyer and the relative bargaining power between the buyer and the seller. Similarly, it can be seen that the procurement cost also fluctuates with the changing equilibrium between the market forces of demand and supply.

The industry context mentioned above has led to this research work on the development of an inventory model for the technology products under uncertain trade credit terms. The model has been solved to illustrate its utility with the help of suitable numerical values of parameters. This research study also incorporates the repeat purchase of newer generation products by the existing adopters of the earlier generation product. Rao and Yamada (1988) emphasized the role of marketing efforts and word of mouth in the diffusion of a product in the repeat purchase of a product. Bass and Bass (2001) proposed an innovation diffusion model for technology generations that allowed repeat purchases. Saito *et al.* (2016) emphasized that positive word of mouth with the existing adopters increases the accessibility of a product in the memory of the non-users. Chen and Fang (2016) studied the factors that influence a consumer's repurchase behavior in the IT industry. It came out of these studies that innovation, as well as the imitation effect, plays a significant role in the repeat purchase, and hence, it needs to be integrated with the demand model.

This research study is structured as follows. Section 1 lays down the motivation behind this research. Section 2 reviews the existing literature about the inventory modeling under imprecise trade credits or imprecise inventory costs. Section 3 pens down the assumptions and the notations to be used in the Model. While the demand model is stated in Section 4, the inventory model is developed in Section 5. Section 6 illustrates the model with the help of a numerical example. Section 7 puts forward the implications of this study for the managers and inventory practitioners. Section 8 concludes this paper while stating the limitations and the scope for future extension of this research

2. LITERATURE REVIEW

In the kind of volatile and unambiguous World that exists today, it is impossible to have access to perfect information; making the traditional deterministic Economic Order Quantity (EOQ) models irrelevant. An

effective way to overcome this challenge is the use of the fuzzy set theory postulated by Zadeh (1965). Zadeh defined the fuzzy set and said that it is associated with a membership function that assigns a grade to each object. We generally make decisions in the real-world environment, where objectives, constraints as well as the results expected from the possible actions are not known precisely (Bellman & Zedah, 1970). The fuzzy set theory and basic ideas of fuzziness have also been introduced by Zimmermann (1976).

One of the earliest works on inventory optimization using fuzzy set theory was by Park (1987) who considered that the cost parameters in the EOQ model are imprecise, and re-examined the traditional model from the perspective of fuzzy systems theory. Vujosevick *et al.* (1996) modeled the EOQ problem using fuzzy logic, assuming that the ordering cost and holding cost are not precisely known. Roy and Maiti (1997) developed a fuzzy EOQ model with warehouse capacity constraints under price-dependent demand and quantity-dependent setup costs. Yao and Lee (1999) expressed the fuzzy lot size as the normal trapezoidal fuzzy number. Mondal and Maiti (2002) used a genetic algorithm to solve the multi-item EOQ model under fuzzy objective and imprecise constraints. Ghomi and Rezaei (2003) took the consumption as a crisp number while considering the holding cost, ordering cost, and selling price to be fuzzy trapezoidal numbers. Mahata and Goswami (2007) took the demand rate, ordering cost, holding cost, and purchase cost as fuzzy numbers and defuzzified the total cost using the graded mean integration representation method.

Mahata and Mahata (2011) developed a fuzzy EOQ model for two levels of trade credit assuming that the retailer is in a powerful position to command full credit from the supplier while offering partial credit to the consumer. Maiti (2011) considered the planning horizon in the inventory model to be imprecise and used a genetic algorithm where the cross-over probability was derived using fuzzy logic. Mahata (2011) developed an EOQ model for gradual non-instantaneous replenishment under trade credits for deteriorating items. Shah *et al.* (2012), and Parvathi and Gajalakshmi (2013) also took ordering cost, holding cost, and lot size as triangular fuzzy numbers and used the graded mean integration representation method for de-fuzzification. Xu (2014) took the demand rate and the deterioration rate as fuzzy variables with known distributions while formulating the inventory optimization model under trade credit. Pattnaik (2013) developed the fuzzy EOQ model for stock-dependent and price-sensitive demand. Soni (2013) developed a fuzzy framework for inventory modeling and coordinated pricing under partial trade credit financing by retailers. Taleizadeh *et al.* (2013) used metaheuristic algorithms like bees colony optimization to solve the fuzzy EOQ Model under pre-payment and quantity discounts. Guchhait *et al.* (2014) considered the demand to be fuzzy and dependent upon the selling price and trade credit. Nagasawa *et al.* (2014) developed an inventory replenishment model for multiple items but did not consider technological substitution. Digalware *et al.* (2014) also used fuzzy logic in supply chain decision-making but in the context of supplier selection.

Chakraborty *et al.* (2015) considered holding cost, procurement cost, ordering cost as well as selling price in the imprecise environment and solved the model using a genetic algorithm to derive the optimal credit period and length of procurement cycle under space and budget constraints. Das *et al.* (2015) considered the credit period offered by the supplier to the manufacturer as a fuzzy variable and optimized the production run time. Mondal *et al.* (2015) developed the EOQ model under two-level partial trade credits while considering the demand to be dependent upon the credit period, amount of credit and inventory level, and inventory costs as fuzzy rough in nature. Roy (2015) considered the uncertain cycle time as a triangular fuzzy number and de-fuzzified the optimal results using the signed distance method. Yadav *et al.* (2015) developed optimal inventory policies for the retailer by taking the opportunity cost and interest rates as fuzzy triangular numbers. Fuzzy profit functions were defined and de-fuzzified using the signed distance method.

Garai *et al.* (2017) derived the possibility, necessity, and credibility measures to determine the chances of occurrence of fuzzy events while modeling inventory for multiple items. Tripathy and Sukla (2018) considered the demand to be a ramp-type function while developing the EOQ model with fuzzy costs. Jaggi *et al.* (2018) fuzzified the demand rate and the deterioration rate as triangular fuzzy numbers and used the Centroid method and signed distance method to de-fuzzify the cost function. Garai *et al.* (2019) used trapezoidal fuzzy numbers to define the time-varying inventory holding cost and the price-dependent demand and developed a fully fuzzy inventory model, treating all the input parameters and decision variables as imprecise. Pramanik and Maiti (2019) proposed a multi-choice artificial bee genetic algorithm to model the inventory for fuzzy promotional efforts. Ohmori and

Yoshimoto (2020) handled the demand uncertainty in inventory control problems using robust optimization.

While the above part of the review discussed the research studies on inventory modeling of single items using fuzzy logic, the coming paragraphs shed light on the recent studies done on the inventory modeling of multiple items under imprecise conditions, with the use of fuzzy logic.

Multiple substitutable items under uncertain demand using the newsvendor approach were considered for inventory modeling in the literature (Shao & Ji, 2006; Dutta & Chakraborty, 2010). Taleizadeh *et al.* (2011a, and 2011b) developed meta-heuristic algorithms to solve single-period inventory problems under uncertain demand. Taleizadeh *et al.* (2009, 2010, 2013a, and 2013b) performed substantial research on constrained optimization of inventory using fuzzy set theory. Garai *et al.* (2016) employed the fuzzy expectation and the possibility/necessity measure to transform the fuzzy model into a deterministic NLP problem. Shekarian *et al.* (2016) investigated the different optimization techniques and algorithms that can be used for optimizing inventory under imprecise conditions. De and Mahata (2019) worked on multiple items with imperfect quality that can be sold as a single batch with a proportionate discount rate. One of the most recent works on substitutable items is that of Maiti (2020) who formulated the fuzzy inventory model for multiple substitutable items being sold at multiple outlets managed by a single entity. Adak and Mahapatra (2020) developed an inventory model for items whose demand and deterioration are dependent upon reliability as well as time.

Table 1 describes the review of EOQ studies on studies done on inventory modeling of multiple items situations using fuzzy logic.

Table 1 Review of the studies on multi-item inventory modelling using fuzzy logic

Membership	Technique	Fuzzy variable	Work	Technology generations?
Linear	GP Technique	Selling period decision variables	Das <i>et al.</i> (2000)	No
		Lot size, Re-order point	Roy and Maiti (1997)	No
	FNLP	Replenishment quantity, sales price, marketing expense	Mondal and Maiti (2002)	No
		time of placing the order	Das <i>et al.</i> (2004)	No
		Lot size, Demand rate	Roy and Maiti (1998)	No
	Conventional Derivation Method	Lot size, lead time	Guchhait <i>et al.</i> (2010)	No
		Lot size, lead time	Wee <i>et al.</i> (2009)	No
Triangular	Conventional Derivation Method	Lot size, shortage amount, Promotional outlays	Baykasoglu and Goken (2011)	No
		Sales price, promotional outlays	Maiti and Maiti (2007)	No
		Lot size, lead time, Re-order point	Xu and Liu (2008)	No
		Binary ordering variable	Yao and Ouyang (2003)	No
	Software	Replenishment quantity, sales price, discount on backlogged sales	Baykasoglu and Goken (2007)	No
		Retailer's lot size	Xie <i>et al.</i> (2006)	No
		Lot size	Huang (2011)	No

Table 1 Review of the studies on multi-item inventory modelling using fuzzy logic (Cont')

Membership	Technique	Fuzzy variable	Work	Technology generations?
Triangular	GA	Lot size, Re-order point, Item-specific area allocation	Jana <i>et al.</i> 2014	No
		Lot size	Chakraborty <i>et al.</i> (2013)	No
		Lot size	Taleizadehet <i>et al.</i> (2013c)	No
	GP Technique	Lot size, shortage amount	Panda <i>et al.</i> (2008)	No
Trapezoidal	PSO	Lot size, lead time	Mousavi <i>et al.</i> (2014)	No
	GRG	advertising frequency, inventory level, item-specific storage area allocation, item-specific storage area allocation	Sahaet <i>et al.</i> (2010)	No
	Conventional Derivation Method	Lot size, lead time, re-order point, safety stock	Wang <i>et al.</i> (2013)	No
Parabolic	GA	Lot size	Chakraborty <i>et al.</i> (2015)	No
	GA	lead time, Replenishment quantity	Maiti (2008)	No
	GA	Lot size	Chakraborty <i>et al.</i> (2015)	No

Abbreviations: GA- Genetic Algorithm, GRG- Generalised Reduced Gradient, PSO- Particle Swarm Optimization, FNLP- Fuzzy Non-Linear Program, GP- Geometric Programming

As shown in **Table 1**, no work has been carried out to date on inventory modeling of multi-generation technology products under fuzzy logic. When it comes to the products with substitutable demand, there is plenty of existing literature on the demand modeling and the inventory modeling of such products. Technology products have their demand influenced by innovation diffusion (Nagpal and Chanda, 2020a). But when it comes to the products with successive technology generations, the literature is very limited. There has been some work by Chanda and Kumar (2017) and Chanda and Kumar (2019) on the inventory modeling for innovation diffusion products, but that has not considered the technology generations. These two research studies considered the innovation products but limited themselves to single generation products only. They have not taken into account the launch of subsequent generations leading to cannibalization of the technology products with advanced products. This is an extension of the earlier studies (Nagpal and Chanda, 2020b; Nagpal and Chanda, 2021) which had developed it as a single period inventory model only, and had taken the credit period and purchase cost to be a fixed parameter. The two areas of extension over the above-mentioned paper are the incorporation of credit-linked demand, the uncertain credit period, and imprecise procurement costs.

3. BUSINESS PROBLEM

The business context in this study is that of a firm that has two successive generations of technology products and has to make inventory decisions for these multiple generations when the demand follows an innovation diffusion pattern, and the procurement cost, as well as trade credits, are imprecise. The procurement cost depends on several factors such as raw material cost, labor factor dynamics, macroeconomic factors, etc that vary with time, and are imprecise.

1.1 Assumptions

The assumptions behind the proposed model are stated below:

1. There are two generations of technology with the first generation being launched at time $t=0$ and the second generation being launched at time $t=\tau$.
2. The demand for the products follows the innovation diffusion and is also influenced by the trade credits and the degree of demand dependent discount on the selling price
3. The credit period and the procurement cost are imprecise
4. The users of the older generation product can switch to the newer generation product
5. The backlogging of the demand is not allowed, and nor are the shortages.
6. The replenishment of both the generations of the product happens jointly

1.2 Notations

List of notations are as follows:

- τ is the point of time at which the product of the second-generation product is launched in the market
- PD_i is the period for which the trade credit is offered by the distribution channel to the customer of the i th generation product
- $\phi_i(t)$ is the fraction of the normal market potential of the i th generation product that adopts it at the time $(t + \Delta t)$ when the trade credits are not offered
- $f_i(t)$ is the fraction of the normal market potential of the i th generation product that adopts it at the time $(t + \Delta t)$ when trade credits are offered
- $F_i(t)$ is the cumulative fraction of the normal market potential of the product of the i th generation that has adopted it till time instant t
- p_i and q_i are the coefficient of innovation and imitation respectively of the product of the i th generation

- $CD_1(t)$ is the cumulative demand of the first generation product till the time t before the introduction of the second generation
- $CD_2(t)$ is the cumulative demand of the second generation product till time t
- M_i is the market potential of the i th generation product when the trade credits are not offered
- I_r is the opportunity cost of credit offered to the consumer (in % terms) by the distribution channel
- I_i is the inventory holding cost as % of the basic purchase cost for the product of i th generation
- C_i be the basic purchase cost per unit for the product of the i th generation
- pr_i is the selling price per unit for the product of the i th generation
- η denotes the sequence of the planning time bucket
- RC_i is the replenishment cost (i.e. ordering cost + inventory carrying cost) for the i th generation product
- TCM_i are the contribution margin per unit (selling price net off basic purchase cost) for the product of the i th generation
- Rev_i is the total revenue for the product of i th generation
- OC_i is the total ordering cost for the product of i th generation
- BC_i is the total basic purchase cost for the product of the i th generation
- HC_i be the total inventory holding cost for the i th generation product
- TP_i is the total profit for the product of i th generation
- A is the fixed non-product-specific ordering cost per order, while A_i is the fixed product-specific ordering cost per order of the i th generation product irrespective of the order volumes
- $\xi_{1\eta}$ is the quantity of the first-generation product ordered in each lot in the time bucket η
- $\xi_{2\eta}$ is the quantity of the second generation product ordered in each lot in the time bucket η
- $\xi_{1\eta'}$ is the quantity of the first generation product ordered in each lot in the time bucket η' after the introduction of the advanced generation product
- ζ is the length of the time bucket for which the inventory norms are fixed

4. DEMAND MODEL

Several credit-linked demand models have been used in the past literature on functional products. The most popular among them is $D(PD) = \theta \exp(\alpha.PD)$ where θ and α are positive constants. This model has been used by many studies, the latest among them being Chern *et al.* (2014) and Wu *et al.* (2017). It has been attempted to integrate this demand function with the innovation diffusion influenced hazard rate as given by Norton and Bass Model (1987) since the demand for technology products is primarily driven by diffusion of innovations.

According to the Norton and Bass Model (1987),

$$f_1(t) = p_1 + q_1 F_1(t) \quad (1)$$

According to Chern *et al.* (2014), if the hazard rate in absence of the credit period is $\phi_1(t)$,

$$f_1(t) = \phi_1(t) \exp(\alpha.PD) \quad (2)$$

$$F_1(t) = \left[\int \phi_1(t) dt \right] \exp(\alpha.PD) \quad (3)$$

From the equations (1), (2), and (3), the following equation (4) is obtained

$$\phi_1(t) \exp(\alpha.PD) = p_1 + q_1 \exp(\alpha.PD) \int \phi_1(t) dt \quad (4)$$

On solving the above-mentioned differential equation, the following equation (5) is obtained

$$\phi_1(t) = \frac{b_1^2 \exp(-b_1 t)}{[p_1 / \exp(\alpha.PD_1)] \{1 + a_1 \exp(-b_1 t)\}^2} \quad (5)$$

And

$$\int \phi_1(t) dt = \frac{1 - \exp(-b_1 t)}{[1 + a_1 \exp(-b_1 t)]} \quad (6)$$

Where $b_1 = p_1 / \exp(\alpha.PD) + q_1$

And $a_1 = q_1 \exp(\alpha.PD) / p_1$

Therefore, for the first generation product, from equations (2) and (5), equation (7) is obtained

$$f_1(t) = \frac{b_1^2 \exp(-b_1 t) [\exp(\alpha.PD_1)]^2}{[p_1 \{1 + a_1 \exp(-b_1 t)\}]^2} \quad (7)$$

From equations (3) and (6), equation (8) is obtained

$$F_1(t) = \frac{[1 - \exp(-b_1 t)] \exp(\alpha.PD_1)}{[1 + a_1 \exp(-b_1 t)]} \quad (8)$$

Similarly, for the second-generation product,

$$f_2(t) = \frac{b_2^2 \exp(-b_2(t-\tau)) [\exp(\alpha.PD_2)]^2}{[p_2 \{1 + a_2 \exp(-b_2(t-\tau))\}]^2} \quad (9)$$

$$F_2(t) = \frac{[1 - \exp(-b_2(t-\tau))] \exp(\alpha.PD_2)}{[1 + a_2 \exp(-b_2(t-\tau))]} \quad (10)$$

If the first generation product is launched at time $t = 0$, and the second generation gets introduced at time $t = \tau$, the demand for the products of the two generations can be written as proposed by Nagpal and Chanda (2020b):

$$\lambda_1(t) = M_1 f_1(t) \text{ for } t \leq \tau \quad (11)$$

$$\lambda_1'(t) = M_1 f_1(t) - M_1 f_1(t) F_2(t - \tau) \text{ for } t \geq \tau \quad (12)$$

$$\lambda_2(t) = M_2 f_2(t) + M_1 f_1(t) F_2(t - \tau) + M_1 F_1(t) f_2(t - \tau) \text{ for } t \geq \tau \quad (13)$$

The above-mentioned equations show that a few of the potential adopters of the first generation technology product will adopt the second generation product instead of the earlier generation product. Also, some existing adopters of the earlier generation product shall buy the second generation product (Bass and Bass, 2001).

5. INVENTORY MODEL IN THE FUZZY CONTEXT

The inventory planning decisions are intermediate-term in nature. This means that the inventory norms are neither fixed for a long time nor changed too frequently to ensure the stability of the system. The time horizons are divided into smaller planning buckets over which the EOQ remains constant and is then, re-worked for the next planning bucket. The distribution channels need to optimize their overall profits over each of these planning buckets.

1.3 Inventory Model in case of single generation existing in the market

The time at which the time bucket η starts is given by $(\eta-1)\zeta$, and the time at which it ends is $(\eta)\zeta$ as shown in Figure 1.

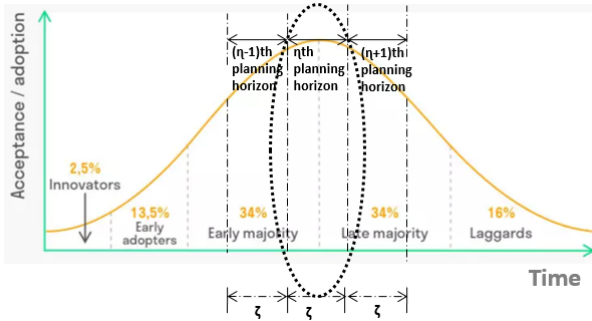


Figure 1 Time buckets into which product life cycle is divided in the single generation case

If the replenishment lot size in this planning time bucket is ξ_{η} , the number of shipments to be delivered in one such period is

$$j = \left(\frac{1}{\xi_{\eta}}\right) [CD_1(\eta, \zeta) - CD_1((\eta-1), \zeta)] \quad (14)$$

The starting time of the k th replenishment cycle is given by

$$t_{k,\eta} = \left(\eta - 1 + \frac{k-1}{j}\right) \zeta \quad (15)$$

The ending time of the k th replenishment cycle is given by

$$t_{(k+1),\eta} = \left(\eta - 1 + \frac{k}{j}\right) \zeta \dots (16)$$

The fuzzy variables \widetilde{C}_1 and \widetilde{PD}_1 are defined as follows:

$$\widetilde{C}_1 = (C_{11}, C_{12}, C_{13}, C_{14}); \quad (C_{11} \geq C_{12} \geq C_{13} \geq C_{14})$$

$$\widetilde{PD}_1 = (PD_{11}, PD_{12}, PD_{13}, PD_{14}); \quad (PD_{11} \leq PD_{12} \leq PD_{13} \leq PD_{14})$$

The inventory economics under the absence of an imprecise business environment are given in the annexures, as taken from earlier research studies (Nagpal and Chanda, 2020b)

Chen's Function Principle (1985) has been used for the operations on fuzzy numbers, and to compute the fuzzy total profit. Thus, the membership function of \widetilde{TP}_1 can be defined as

$$\widetilde{TP}_1 = (TP_{11}, TP_{12}, TP_{13}, TP_{14}).$$

$$\text{Where } \widetilde{TP}_{1t} = \widetilde{TCM}_{1t} - \widetilde{RC}_{1t} - \widetilde{CC}_{1t} \quad (17)$$

Applying fuzzy logic terminology to the equations A1.1 to A1.7 in the annexure, the equations (17a) to (17f) are obtained.

$$\widetilde{TCM}_{1t} = \widetilde{Rev}_{1t} - \widetilde{BC}_{1t} \quad (17a)$$

$$\widetilde{Rev}_{1t} = pr_1 \cdot \sum_{k=1}^{k=\widetilde{f}_i} \int_{t=\widetilde{t}_{k,\eta}}^{t=\widetilde{t}_{(k+1),\eta}} \widetilde{\lambda}_{1t}(t) \cdot dt \quad (17b)$$

$$\widetilde{BC}_{1t} = \widetilde{C}_{1t} \cdot \sum_{k=1}^{k=\widetilde{f}_i} \int_{t=\widetilde{t}_{k,\eta}}^{t=\widetilde{t}_{(k+1),\eta}} \widetilde{\lambda}_{1t}(t) \cdot dt \quad (17c)$$

$$\widetilde{RC}_{1t} = \widetilde{f}_i(A + A_1) + \sum_{k=1}^{k=\widetilde{f}_i} \widetilde{HC}_{1t} \quad (17d)$$

$$\widetilde{CC}_{1t} = I_r \cdot \widetilde{C}_{1t} \cdot \widetilde{PD}_{1t} \cdot \int_{t=\widetilde{t}_{k,\eta}}^{t=\widetilde{t}_{(k+1),\eta}} \widetilde{\lambda}_{1t}(t) dt \quad (17e)$$

$$\widetilde{HC}_{1t} = I_1 \widetilde{C}_{1t} \int_{t=\widetilde{t}_{k,\eta}}^{t=\widetilde{t}_{(k+1),\eta}} \int_t^{\widetilde{t}_{(k+1),\eta}} \widetilde{\lambda}_{1t}(t) dt dt \quad (17f)$$

Applying fuzzy logic terminology to the equations (14), (15), and (16), one can obtain equations 18a, 18b, and 18c.

$$\widetilde{t}_{k,\eta} = \left(\eta - 1 + \frac{k-1}{\widetilde{f}_i}\right) \zeta \quad (18a)$$

$$\widetilde{t}_{(k+1),\eta} = \left(\eta - 1 + \frac{k}{\widetilde{f}_i}\right) \zeta \quad (18b)$$

$$\widetilde{f}_i = \left(\frac{1}{\xi_{\eta+1}}\right) [\widetilde{CD}_{1t}(\eta, \zeta) - \widetilde{CD}_{1t}((\eta-1), \zeta)] \quad (18c)$$

Similarly, applying fuzzy logic to equations (11), (12), and (13), the equations (19a), (19b), and (19c) are obtained

$$\widetilde{\lambda}_{1t}(t) = M_1 \widetilde{f}_{1t}(t) \text{ for } t \leq \tau \quad (19a)$$

$$\widetilde{\lambda}_{1t}(t) = M_1 \widetilde{f}_{1t}(t) - M_1 \widetilde{f}_{1t}(t) \widetilde{F}_{2t}(t - \tau) \text{ for } t \geq \tau \quad (19b)$$

$$\widetilde{\lambda}_{2t}(t) = M_2 \widetilde{f}_{2t}(t) + M_1 \widetilde{f}_{1t}(t) \widetilde{F}_{2t}(t - \tau) + M_1 \widetilde{F}_{1t}(t) \widetilde{f}_{2t}(t - \tau) \text{ for } t \geq \tau \quad (19c)$$

Similarly, applying fuzzy logic to equations (7), (8), (9), and (10), the equations (20a) to (20f) are obtained.

$$\widetilde{f}_{1t}(t) = \frac{\widetilde{b}_{1t}^2 \exp(-\widetilde{b}_{1t}t) [\exp(\alpha \cdot \widetilde{PD}_{1t})]^2}{[p_1 \{1 + \widetilde{a}_{1t} \cdot \exp(-\widetilde{b}_{1t}t)\}^2]} \quad (20a)$$

$$\widetilde{F}_{1t}(t) = \frac{[1 - \exp(-\widetilde{b}_{1t}t)] \exp(\alpha \cdot \widetilde{PD}_{1t})}{[1 + \widetilde{a}_{1t} \cdot \exp(-\widetilde{b}_{1t}t)]} \quad (20b)$$

$$\widetilde{f}_{2t}(t) = \frac{\widetilde{b}_{2t}^2 \exp(-\widetilde{b}_{2t}(t-\tau)) [\exp(\alpha \cdot \widetilde{PD}_{2t})]^2}{[p_2 \{1 + \widetilde{a}_{2t} \cdot \exp(-\widetilde{b}_{2t}(t-\tau))\}]^2} \quad (20c)$$

$$\widetilde{F}_{2t}(t) = \frac{[1 - \exp(-\widetilde{b}_{2t}(t-\tau))] \exp(\alpha \cdot \widetilde{PD}_{2t})}{[1 + \widetilde{a}_{2t} \cdot \exp(-\widetilde{b}_{2t}(t-\tau))]} \quad (20d)$$

$$\widetilde{b}_{1t} = p_1 / \exp(\alpha \cdot \widetilde{PD}_{1t}) + q_1 \quad (20e)$$

$$\widetilde{a}_{1t} = q_1 \exp(\alpha \cdot \widetilde{PD}_{1t}) / p_1 \quad (20f)$$

Using the Median rule of defuzzification method, the total profit in the planning bucket η , i.e. “ TP_{1m} ” can be given as:

$$TP_{1m} = \left(\frac{1}{4}\right) \left[\sum_{i=1}^4 \widetilde{TP}_{1t} \right]$$

The objective function is stated as under:

$$\text{Max. } TP_{1m} = \left(\frac{1}{4}\right) \left[\sum_{i=1}^4 \widetilde{TP}_{1t} \right]$$

Subject to all \widetilde{f}_i being positive integers

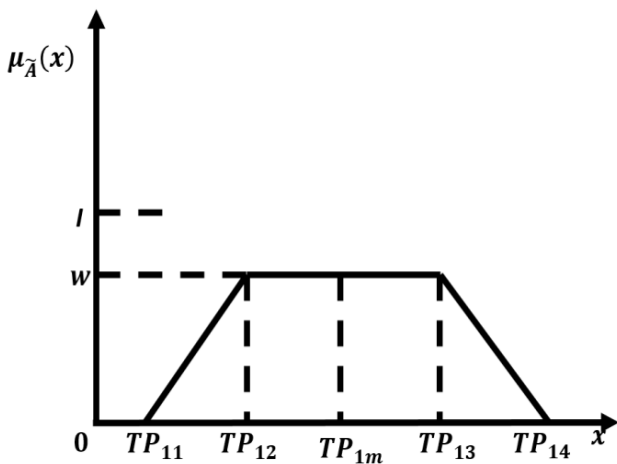


Figure 2 De-fuzzifying the Total Profit Function by Median Rule

Figure 2 represents the median rule of defuzzification for the total profit function.

1.4 Inventory Model for the two generations scenario

After the second generation product has been launched, let the planning time bucket η' start at time $t = \tau + (\eta' - 1)(\zeta)$ and end at time $t = \tau + (\eta')(\zeta)$. This is illustrated in **Figure 3**.

If $\xi_{1\eta'}$ and $\xi_{2\eta'}$ be the order quantities (under credit periods PD_{11} and PD_{21} of the products of the first generation and the second generation respectively, the number of orders to be placed for the products is:

$$j_1' = [CD_1'(\tau + (\eta)(\zeta)) - CD_1'(\tau + (\eta - 1)(\zeta))] / \xi_{1\eta'} \quad (21)$$

$$j_2 = [CD_2(\tau + (\eta)(\zeta)) - CD_2(\tau + (\eta - 1)(\zeta))] / \xi_{2\eta} \quad (22)$$

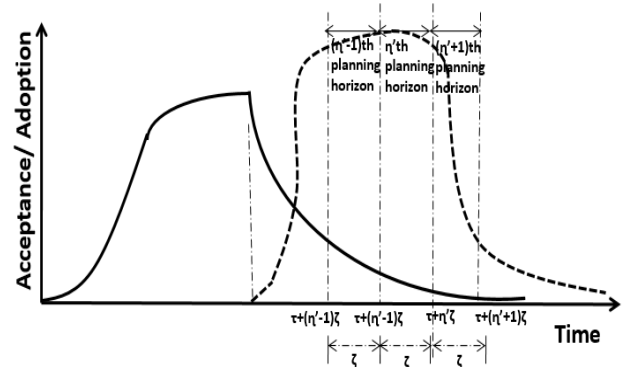


Figure 3 Time buckets into which product life cycle is divided after the launch of the second generation product

where $CD_1'(t)$ is the cumulative demand of the first generation product till the time instant t after the second generation product is launched in the market

The starting time for the k th replenishment cycle for the first generation is $t_{(k-1),\xi_{1\eta'}} = \tau + (\eta - 1 + (k - 1)/j_1')\zeta$ and the ending time is

$$t_{k,\xi_{1\eta'}} = \tau + (\eta - 1 + k/j_1')\zeta \quad (23)$$

Similarly, the starting time for the k th replenishment cycle for the second generation is $t_{(k-1),\xi_{2\eta}} = \tau + (\eta - 1 + (k - 1)/j_2)\zeta$ and the ending time is

$$t_{k,\xi_{2\eta}} = \tau + (\eta - 1 + k/j_2)\zeta \quad (24)$$

When the logistics for both generations of products are pooled to reap the operational synergies, we can say that $t_{(k-1),\xi_{1\eta'}} = t_{(k-1),\xi_{2\eta}}$; and $t_{k,\xi_{1\eta'}} = t_{k,\xi_{2\eta}}$

The inventory economics have been stated in the annexure in the absence of an imprecise environment as given by Nagpal and Chanda (2020b).

Using Chen's Function Principle (1985) (as in the case of single generations scenario earlier in the paper), the membership function of \widetilde{TP}' is defined as

$$\widetilde{TP}' = (TP_1', TP_2', TP_3', TP_4').$$

Where

$$\widetilde{TP}_i' = \widetilde{TCM}_i' - \widetilde{RC}_i' - \widetilde{CC}_i'$$

Applying fuzzy logic terminology to the equations A3.1 to A3.5 in the annexure, one can obtain the equations (25a) to (25f).

$$\widetilde{TCM}_i' = \widetilde{Rev}_i' - \widetilde{BC}_i' \quad (25a)$$

$$\widetilde{Rev}_i' = pr_1 \cdot \sum_{k=1}^{\widetilde{f}_{1i}'} \int_{t=\widetilde{t}_{k,\eta 1i}}^{t=\widetilde{t}_{(k+1),\eta 1i}} \widetilde{\lambda}_{1i}'(t) \cdot dt + pr_2 \cdot \sum_{k=1}^{\widetilde{f}_{2i}'} \int_{t=\widetilde{t}_{k,\eta 2i}}^{t=\widetilde{t}_{(k+1),\eta 2i}} \widetilde{\lambda}_{2i}'(t) \cdot dt \quad (25b)$$

$$\widetilde{BC}_i' = \widetilde{C}_{1i} \cdot \sum_{k=1}^{\widetilde{f}_{1i}'} \int_{t=\widetilde{t}_{k,\eta 1i}}^{t=\widetilde{t}_{(k+1),\eta 1i}} \widetilde{\lambda}_{1i}'(t) \cdot dt + \widetilde{C}_{2i} \cdot \sum_{k=1}^{\widetilde{f}_{2i}'} \int_{t=\widetilde{t}_{k,\eta 2i}}^{t=\widetilde{t}_{(k+1),\eta 2i}} \widetilde{\lambda}_{2i}'(t) \cdot dt \quad (25c)$$

$$\widetilde{RC}_i' = \widetilde{J}_{1i}'(A + A_1) + \sum_{k=1}^{\widetilde{f}_{1i}'} \widetilde{HC}_i' \quad (25d)$$

$$\widetilde{CC}_i' = I_r \cdot \widetilde{C}_{1i} \cdot \widetilde{PD}_{1i} \cdot \int_{t=\widetilde{t}_{k,\eta 1i}}^{t=\widetilde{t}_{(k+1),\eta 1i}} \widetilde{\lambda}_{1i}'(t) dt + I_r \cdot \widetilde{C}_{2i} \cdot \widetilde{PD}_{2i} \cdot \int_{t=\widetilde{t}_{k,\eta 2i}}^{t=\widetilde{t}_{(k+1),\eta 2i}} \widetilde{\lambda}_{2i}'(t) \cdot dt \quad (25e)$$

$$\widetilde{HC}_i' = I_1 \widetilde{C}_{1i} \int_{t=\widetilde{t}_{k,\eta 1i}}^{t=\widetilde{t}_{(k+1),\eta 1i}} \int_t^{\widetilde{t}_{(k+1),\eta 1i}} \widetilde{\lambda}_{1i}'(t) dt dt + I_2 \widetilde{C}_{2i} \int_{t=\widetilde{t}_{k,\eta 2i}}^{t=\widetilde{t}_{(k+1),\eta 2i}} \int_t^{\widetilde{t}_{(k+1),\eta 2i}} \widetilde{\lambda}_{2i}'(t) dt dt \quad (25f)$$

Applying fuzzy terminology to equations (20 and 21), the equations (26a) to (26d) can be obtained.

$$\widetilde{t}_{(k),\eta 1i} = \left(\eta - 1 + \frac{k-1}{\widetilde{J}_1'} \right) \zeta \quad (26a)$$

$$\widetilde{t}_{(k+1),\eta 1i} = \left(\eta - 1 + \frac{k}{\widetilde{J}_1'} \right) \zeta \quad (26b)$$

$$\widetilde{t}_{k,\eta 2i} = \left(\eta - 1 + \frac{k-1}{\widetilde{J}_2} \right) \zeta \quad (26c)$$

$$\widetilde{t}_{(k+1),\eta 2i} = \left(\eta - 1 + \frac{k}{\widetilde{J}_2} \right) \zeta \quad (26d)$$

Applying fuzzy terminology to equations (18) and (19), the equations (27a) and (27b) are obtained.

$$\widetilde{J}_{1i}' = \left(\frac{1}{\xi_{1\eta 1i}'} \right) [\widetilde{CD}_{1i}'(\eta, \zeta) - \widetilde{CD}_{1i}'((\eta - 1), \zeta)] \quad (27a)$$

$$\widetilde{J}_{2i} = \left(\frac{1}{\xi_{2\eta 2i}} \right) [\widetilde{CD}_{2i}(\eta, \zeta) - \widetilde{CD}_{2i}((\eta - 1), \zeta)] \quad (27b)$$

Using the Median rule of defuzzification method, the total profit in the planning bucket η , i.e. “ TP_{1m} ” can be given as:

$$TP_m' = \left(\frac{1}{4} \right) \left[\sum_{i=1}^4 \widetilde{TP}_i' \right]$$

The objective function is stated as under:

$$Max TP_m' = \left(\frac{1}{4} \right) \left[\sum_{i=1}^4 \widetilde{TP}_i' \right]$$

Subject to all \widetilde{J}_{1i}' and \widetilde{J}_{2i} being positive integers

$$\text{And } \widetilde{J}_{1i}' = \widetilde{J}_{2i}$$

Theorem 1: The uncertain nature of the basic purchase cost results in an increase in the optimal replenishment quantity.

Proof: For a given value of trade credit in any planning horizon, the demand is constant, let us say, D. So, the EOQ can be stated as $EOQ = \sqrt{\left(\frac{2(O+O_1)D}{H} \right)} = \sqrt{\left(\frac{2(O+O_1)D}{(I_1\zeta + I_r \cdot PD_1) C_1} \right)}$
 $\frac{\partial(EOQ_m)}{\partial C_1} = -(0.5) \sqrt{\left(\frac{2(O+O_1)D}{(I_1\zeta + I_r \cdot PD_1) C_1} \right)} (C_1)^{-1.5}$ which is always negative.

$\frac{\partial^2(EOQ_m)}{\partial^2 C_1} = (0.75) \sqrt{\left(\frac{2(O+O_1)D}{(I_1\zeta + I_r \cdot PD_1) C_1} \right)} (C_1)^{-2.5}$ which is always positive.

That means the fall in EOQ with an increase in basic purchase cost by x% is lower than the rise in EOQ by a decrease in basic purchase cost by x%. Thus, the variability of the basic purchase cost results in a fall in the EOQ.

Theorem 2: With the incorporation of repeat purchase in the Model, the optimal replenishment quantity of the second generation product increases.

Proof: The repeat purchase leads to an increase in the demand rate of the second generation product, ensuring faster depletion of inventories for a given lot size and therefore, lesser holding costs relative to the ordering costs. This phenomenon is illustrated in **Figure 4**.

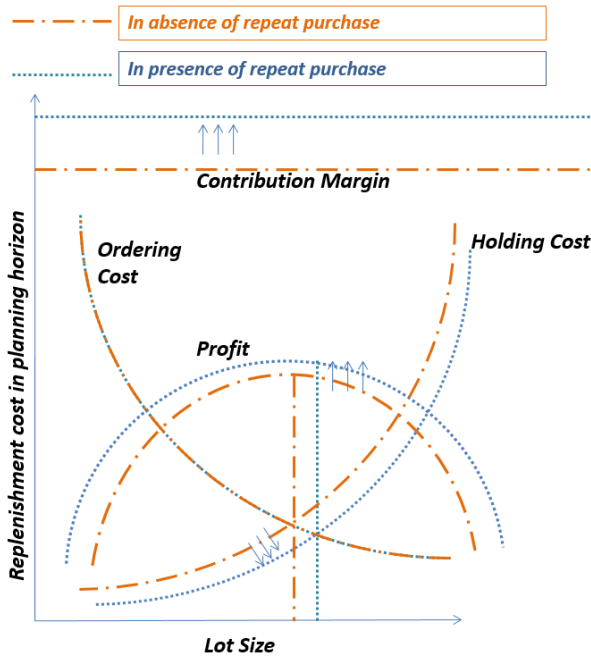


Figure 4 Influence of the repeat purchase (of the advanced product by the existing adopters of older product) on the optimal lot size of the advanced product

This tends to increase the EOQ for the second generation product, and shifting of the point of maxima in the total cost curve to the right.

Theorem 3: As the tendency of the customer to innovate and imitate increases, the optimal replenishment quantity of the technology products increases in the earlier phases of time and falls in the later phases.

Proof: With the increase in innovation and imitation, the adoption rate increases further, leading to higher demand in the initial phase and lower demand in the later phase of time. This results in faster (and lower) depletion of inventories in the earlier (and later) phase of time, leading to an increase (and decrease) in EOQ in the initial (and later) period. This is illustrated in **Figure 5**.

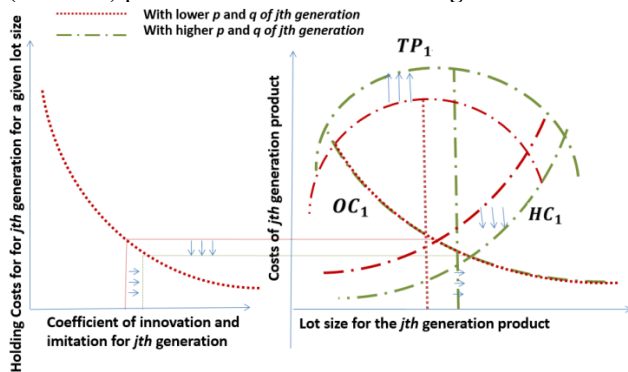


Figure 5 Influence of innovating and imitating tendency of the consumer on EOQ for technology products

6. Numerical Illustrations

The numerical illustrations are performed by assigning certain values to the parameters as follows.

$$A = 1000000,$$

$$A_1 = 200000,$$

$$A_2 = 200000,$$

$$I_1 = .15, I_2 = .15, M_1 = 100000,$$

$$M_2 = 200000,$$

$$p_1 = .05, q_1 = .25, p_2 = .06, q_2 = .4, I_r = 0.18,$$

$$pr_1 = 3500,$$

$$pr_2 = 4500,$$

$$C_{11} = 1500,$$

$$C_{11} = 1600,$$

$$C_{13} = 1700,$$

$$C_{14} = 1800,$$

$$C_{21} = 1700,$$

$$C_{22} = 1800,$$

$$C_{23} = 1900,$$

$$C_{24} = 2000,$$

$$\alpha = .15, \tau = 0.5,$$

$$\varepsilon = 0.5,$$

$$PD_{11} = .5,$$

$$PD_{12} = .4,$$

$$PD_{13} = .3,$$

$$PD_{14} = .2,$$

$$PD_{21} = .5,$$

$$PD_{22} = .4,$$

$$PD_{23} = .25,$$

$$PD_{24} = .25$$

Although these values are hypothetical answers, they are assumed with the input of the practitioners in the supply chain of smartphones and tablets. Therefore, each of the parameter values is a realistic number from the context of the economics of production, logistics, trade financing, and diffusion aspects of innovation products.

First, the single generation scenario is run considering the absence of the second generation product. The following results can be obtained as shown in **Table 2**.

Table 2 The optimal EOQ under a single generation scenario for four planning horizons

Time period	EOQ ₁ ('000 units)	Rev' (Mn INR)	TP' (Mn INR)
$t = 0 \text{ to } 0.5$	435.7	1525	712
$t = .5 \text{ to } 1.0$	466.2	1631	767
$t = 1 \text{ to } 1.5$	211.7	741	343
$t = 1.5 \text{ to } 2$	47.5	298	102

As it can be observed, the optimal EOQ increases initially due to the rising sales volumes, and then starts declining. This is because of the higher demand initially caused by the innovation effect of the advertising, which leads to the faster attainment of the maturity stage in the product life cycle. Whenever any technology product gets launched, it experiences the growth stage (after the introduction stage) in which the sales are quite high. The innovators adopt the product in the early stages as they are willing to take the risk and try the product. Over a while, the imitators contribute majorly to the sales; but every product has a market potential ceiling. Therefore, the product reaches the maturity stage and decline stage, leading to a drop in sales.

Then, the two generations model is run. On creating an optimization code in Matlab and running the same, the following results can be obtained as captured in **Table 3**.

Table 3 The optimal EOQ post the launch of the second-generation product with repeat purchase of the second generation by the existing adopters of first-generation

Period	EOQ ₁ ('000 units)	EOQ ₂ ('000 units)	Rev' (Mn INR)	TP' (Mn INR)
$t = .5 \text{ to } 1$	81.4	838.7	16235	8867
$t = 1 \text{ to } 1.5$	5.6	459.3	6259	3420
$t = 1.5 \text{ to } 2$	0.8	128.0	1158	611

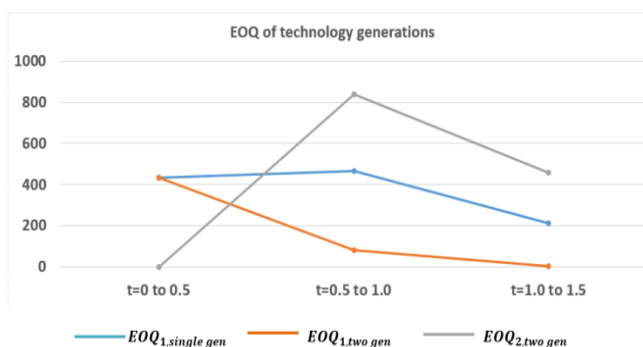


Figure 6 The EOQ behavior of the technology generations over the first three time horizons

In comparing **Table 2** and **Table 3** (as shown in **Figure 6** and **Figure 7**), it can be discovered that with the launch of the second-generation product, the EOQ of the

first generation product falls due to cannibalization of a fraction of its demand by the second generation product. This has been observed in real situations, such as the demand for i5 processors declining with the launch of i7 processors.

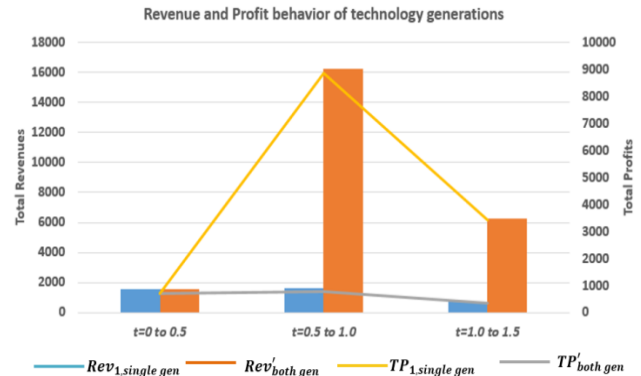


Figure 7 The revenues and profit behavior of the technology generations over the time horizons

Table 4 captures the optimal replenishment quantities for both generations if there were no repeat purchases. In comparing **Table 3** and **Table 4**, it can be discovered that the EOQ of the second-generation product is much lesser when the repeat purchase by the existing adopters of the earlier generation product is not allowed. The reduction in demand induced by this leads to the fall in EOQ. This is often observed in the supply chains or distribution channels that the flow of the earlier generation product does can be sustained if the switching of the product by the existing users of the earlier generation product is not allowed.

Table 4 The optimal EOQ post the launch of the second generation product without repeat purchase of the second generation by the existing adopters of the first-generation product

Time period	EOQ ₁ ('000 units)	EOQ ₂ ('000 units)	Rev' (Mn INR)	TP' (Mn INR)
$t = .5 \text{ to } 1$	81.4	421.3	8723	4442
$t = 1 \text{ to } 1.5$	5.6	435.3	5935	3229
$t = 1.5 \text{ to } 2$	0.3	392.2	4281	2437

Then, the sensitivity analysis of the model is carried out with the change in the innovation and imitation coefficients. The innovation coefficient and the imitation coefficient are increased by 20% (which allows the products to spread faster among the market potential), and then the same steps for modeling the optimal replenishment quantity are followed as mentioned above. The results are displayed in **Table 5** for both the scenarios-with and without the repeat purchase.

In comparing **Table 5** with **Table 3** and **Table 4**, the optimal EOQ of the second generation product rises with

the increase in innovation and imitation effect with the faster diffusion caused by an increase in coefficients.

Table 5 The optimal EOQ with the increase in the innovation and imitation coefficients by 20%, with and without allowing the repeat purchase

Repeat purchase?	EOQ ₁ (^{'000} units)	EOQ ₂ (^{'000} units)	Rev' (Mn INR)	TP' (Mn INR)
Yes	78.6	1017.3	19410	10629
No	78.6	498.5	10073	5458

7. MANAGERIAL IMPLICATIONS

The products in the modern world undergo technology improvements at a fast speed, and the newer generations are launched after regular intervals to address the needs of the consumers better. In such cases, it is not only the non-linear and non-stationary demand rate that poses a challenge but also the ever-changing business environment in which many variables are ambiguous, which also adds to the complexity. The procurement cost also changes at times because of the varying commodity prices, political conditions, geographical constraints, changing regulations, changing demand-supply equilibriums, etc. And the trade credits may also change with the change in the relative bargaining power of the parties involved, or with the changing financing limitations.

It is in this context of an imprecise environment that this study has many important implications for managerial decision-making. First and foremost, it lays down how the imprecise situations of the real world can be converted into decision-making problems using fuzzy logic. It becomes very important for the managers of today's dynamic and unpredictable business situations to handle the uncertainty by quantifying the same, for which the fuzzy set theory proves to be of great help.

Second, it can be observed that there is a significant jump in the overall volumes with the launch of the advanced generation of a product, contingent on the additional utility brought by the newer product to the consumer in terms of function and features. The revenue and the total profit increase considerably with the launch of the second generation product since that has higher market potential and faster diffusion as compared to that of the first generation by having more features. This leads to an increase in the overall market, and hence, better inventory replenishment efficiencies. The industry practitioners and product planners need to leverage this learning and keep coming up with innovative features in the products to improve the overall volumes and operational productivity.

Third, the optimal replenishment quantity of the technology products is higher in the initial periods and then falls in the later periods. This is because the volumes in the case of technology products are very high in the initial time zones, which leads to a higher optimal replenishment quantity. Fourth, the EOQ for the second generation product is higher than that of the earlier generation products. This is because the second-generation product is

a better version of the earlier generation product, and hence, enjoys a higher demand rate, leading to an increase in the EOQ.

Fifth, it needs to be remembered that the order quantity derived in this model is not static but varies from one planning period to another. This is because the lifecycle dynamics of the product have been taken care of, as the product lifecycle is quite short in the case of technology generations. This is a very important fact to be considered by inventory managers dealing with innovation products (whose demand is governed by theory of diffusion of innovations), since they otherwise tend to keep a constant replenishment quantity over time.

Sixth, this work can also help the sellers of high technology products, who have a buyer with a history of imprecise payment timings, in determining the length of the credit period that they can offer to the buyer. While too small a credit period may result in reduction of sales volumes, too large of it may be detrimental to the working capital needs of the seller. Therefore, the seller needs to offer an optimal trade credit period to the buyer. Finally, when the procurement costs are uncertain in the dynamic demand-supply equilibrium, this model will help the inventory practitioners to make a sound business decision and strike the right balance between the pessimistic situation and the optimistic situation.

8. CONCLUSIONS AND FUTURE DIRECTIONS

This research paper has used the fuzzy set theory to model and optimize inventory decisions for multi-generational technology products under imprecise procurement costs and trade credits. The fuzzy framework discussed here can be used to model the inventory decisions in the industry when any business variable is imprecise.

There are a few limitations of the study. First, it has considered only two technology generations. Second, it has assumed that the second generation product has higher market potential than the first generation product, while it may not be true always. Third, the backlogging of demand has not been considered to make it relatively simpler to understand. Fourth, the deterministic demand rate has been taken in this paper, while it may have an element of stochasticity in the industry situations. There are a few extensions in which further work can happen in the future. Fifth, every repeat purchase affects the environment as the earlier product gets disposed. Such environmental cost has not been taken in the study.

This research study can be extended in several directions. First, the full or partial backlogging of the demand and the product shortages can be allowed. This is because, at times, it may be more economical for the seller to make the buyer wait for some time and get the product later. Second, the demand pattern can be considered to be stochastic, and the safety stock provisioning for a certain service level can be done in the model. This is because it is not economical to meet 100% demand in uncertain demand conditions. Third, the environmental cost of the e-waste produced due to the repeat purchase of electronic products can be quantified and considered in the model. This becomes very important in today's context of increasing

thrust on business sustainability. Fourth, future research can consider the influence of the financial derivatives that can be used to hedge the increase in procurement costs or credit rate of interest. Fifth, the influence of the receivables trading in making the credit period decisions can be considered as an extension of this work, since the trading of receivables is a very common practice in the high growth organizations that have plenty of lucrative investment opportunities and would like to rotate their funds faster.

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APPENDIX 1: INVENTORY ECONOMICS FOR SINGLE GENERATION

The inventory carrying cost in the k th replenishment cycle in the planning time bucket η are given by

$$HC_1 = I_1 C_1 \int_{t_{k,\eta}}^{t_{(k+1),\eta}} I_1(t) dt = I_1 C_1 \int_{t=t_{k,\eta}}^{t=t_{(k+1),\eta}} \int_t^{t_{(k+1),\eta}} \lambda_1(t) dt dt \quad (A1.1)$$

The trade-credit cost in the k th ordering cycle of the planning time bucket η is given as

$$CC_1 = I_r \cdot C_1 \cdot PD_1 \cdot \int_{t=t_{k,\eta}}^{t=t_{(k+1),\eta}} \lambda_1(t) dt \quad (A1.2)$$

The replenishment costs (sum of ordering and inventory carrying costs) for the η time bucket are given as

$$RC_1 = j(A + A_1) + \sum_{k=1}^{k=j} HC_1 \quad A1.3$$

The revenue is given by the product of price and demand

$$Rev_1 = pr_1 \cdot \sum_{k=1}^{k=j} \int_{t=t_{k,\eta}}^{t=t_{(k+1),\eta}} \lambda_1(t) \cdot dt \quad (A1.4)$$

$$BC_1 = C_1 \cdot \sum_{k=1}^{k=j} \int_{t=t_{k,\eta}}^{t=t_{(k+1),\eta}} \lambda_1(t) \cdot dt \quad (A1.5)$$

$$TCM_1 = Rev_1 - BC_1 \quad (A1.6)$$

$$TP_1 = TCM_1 - RC_1 - CC_1 \quad (A1.7)$$

APPENDIX 2: FUZZY DECISION VARIABLES

By the model assumptions, the basic purchase cost per unit (C_1) and credit period (PD_1) are imprecise, and thus treated as fuzzy variables and represented as \widetilde{C}_1 and \widetilde{PD}_1 . Therefore, the total Profit in the planning bucket TP_1 can be stated in the form of fuzzy sets. The fuzzy profit per unit time \widetilde{TP}_1 can be defined as follows:

$$\widetilde{TP}_1 = \widetilde{Rev}_1 - \widetilde{BC}_1 - \widetilde{RC}_1 - \widetilde{CC}_1 \quad (A2.1)$$

The Trapezoidal membership function has been deployed for fuzzy variables \widetilde{C}_1 and \widetilde{PD}_1 . Let \widetilde{C}_1 is a trapezoidal fuzzy number, $\widetilde{C}_1 = (c, a, b, d)$, where a, b, c, d are real numbers and $c \leq a \leq b \leq d$.

The Trapezoidal membership function $\mu_{\widetilde{A}}(x)$ for the Fuzzy numbers can be described as follows and is graphically defined in **Figure A2**.

$$\mu_{\widetilde{A}}(x) = \begin{cases} w \frac{x-c}{a-c}; & c \leq x \leq a \\ w; & a \leq x \leq b \\ w \frac{x-d}{b-d}; & b \leq x \leq d \\ 0; & \text{otherwise} \end{cases}$$

Where, $0 < w \leq 1$

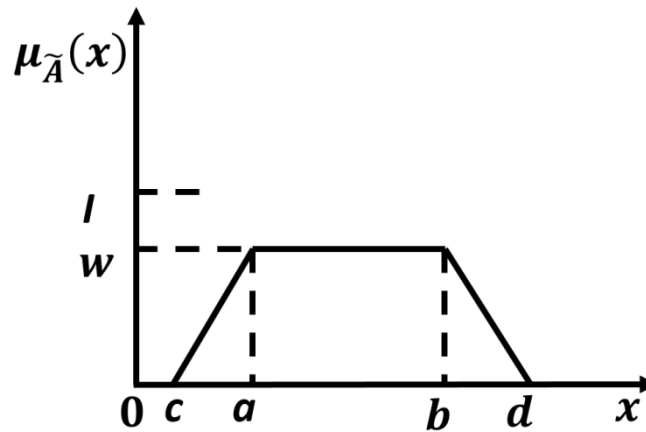


Figure A2 Visual Representation of the fuzzy number

APPENDIX 3: INVENTORY ECONOMICS IN CASE OF A TWO-GENERATION SCENARIO WHEN THERE ARE NO FUZZY VARIABLES

Let RC' , CC' , BC' , OC' , HC' , Rev' , TCM' , TP' be the total replenishment cost (ordering + holding), credit cost, basic purchase cost, ordering cost, holding cost, Revenue, Total contribution margin, and Total Profit respectively for both the generations of products combined in a planning time bucket post the launch of the second generation product

$$RC' = RC_1 + RC_2 = j'_1 A_1 + j_2 A_2 + H_1 C_1 \sum_{k=1}^{k=j'_1} \int_{t=t_{k,\eta_1}}^{t=t_{(k+1),\eta_1}} \int_t^{t_{(k+1),\eta_1}} \lambda'_1(t) dt dt + H_2 C_2 \sum_{k=1}^{k=j_2} \int_{t=t_{k,\eta_2}}^{t=t_{(k+1),\eta_2}} \int_t^{t_{k,\eta_2}} \lambda_2(t) dt dt \quad (A3.1)$$

$$CC' = CC_1 + CC_2 = I_r \cdot C_1 \cdot PD_1 \cdot \sum_{k=1}^{k=j'_1} \int_{t=t_{k,\eta_1}}^{t=t_{(k+1),\eta_1}} \lambda'_1(t) dt + I_r \cdot C_2 \cdot PD_2 \cdot \sum_{k=1}^{k=j_2} \int_{t=t_{k,\eta_2}}^{t=t_{(k+1),\eta_2}} \lambda_2(t) dt \quad (A3.2)$$

$$Rev' = Rev_1 + Rev_2 = pr_1 \cdot \sum_{k=1}^{k=j'_1} \int_{t=t_{k,\eta_1}}^{t=t_{(k+1),\eta_1}} \lambda'_1(t) \cdot dt + pr_2 \cdot \sum_{k=1}^{k=j_2} \int_{t=t_{k,\eta_2}}^{t=t_{(k+1),\eta_2}} \lambda_2(t) \cdot dt \quad (A3.3)$$

$$BC' = BC_1 + BC_2 = C_1 \cdot \sum_{k=1}^{k=j'_1} \int_{t=t_{k,\eta_1}}^{t=t_{(k+1),\eta_1}} \lambda'_1(t) \cdot dt + C_2 \cdot \sum_{k=1}^{k=j_2} \int_{t=t_{k,\eta_2}}^{t=t_{(k+1),\eta_2}} \lambda_2(t) \cdot dt \quad (A3.4)$$

$$TP' = Rev' - BC' - RC' - CC' \quad (A3.5)$$

The optimization problem is the maximization of the total profit and can be stated as

Max. $TP' = Rev' - BC' - RC'$

Subject to the constraints

j'_1, j_2 are positive integers

$$j'_1 = j_2$$

APPENDIX 4: Fuzzy decision variables in case of two generation scenario

Since the basic purchase cost per unit C_2 and credit period PD_2 , being imprecise by model assumptions, can be represented fuzzy variables as \widetilde{C}_2 and \widetilde{PD}_2 . Therefore, the fuzzy total Profit in the planning bucket TP' can be defined as follows:

$$\widetilde{TP'} = \widetilde{Rev'} - \widetilde{BC'} - \widetilde{RC'} - \widetilde{CC'} \quad (A3.6)$$

Let $\widetilde{C}_1, \widetilde{C}_2, \widetilde{PD}_1$ and \widetilde{PD}_2 are the trapezoidal fuzzy numbers with the known relative order of magnitude. The new fuzzy variables introduced here \widetilde{C}_2 and \widetilde{PD}_2 are defined as follows:

$$\begin{aligned} \widetilde{C}_2 &= (C_{21}, C_{22}, C_{23}, C_{24}); \quad (C_{21} \geq C_{22} \geq C_{23} \geq C_{24}) \\ \widetilde{PD}_2 &= (PD_{21}, PD_{22}, PD_{23}, PD_{24}); \quad (PD_{21} \leq PD_{22} \leq PD_{23} \leq PD_{24}) \end{aligned}$$

Dr. Gaurav Nagpal is an Assistant Professor in the Management Group (Off-Campus) at Birla Institute of Technology and Science Pilani. He received his Ph.D. from BITS Pilani and is a full-time MBA from IIFT Delhi. He has also done EPMBD (one-year executive program in managing business decisions) from IIM Calcutta and is CSCP qualified from APICS. He was a 99.83 percentile scorer in the GATE exam in the year 2004. He cleared the Indian Engineering Services exam (conducted by UPSC) in 2006. He is also UGC NET Qualified, and CFA L-1. He has research interests in data analytics, operations research, operations management, and supply chain management. He has published 11 research articles in SCI/ ABDC/ Scopus/ Web of Science journals.

Dr. Udayan Chanda is currently working as Associate Professor in Department of Management, Birla Institute of Technology & Science (BITS) Pilani. Earlier he was associated with Industrial Statistics Lab., Department of Information & Industrial Engineering Yonsei University as Post Doctoral Fellow and Department of Operational Research, University of Delhi as Assistant Professor (Ad-hoc). He received his Ph.D. degree in Marketing Models and Optimization (Operational Research) from University of Delhi, Delhi. He has published numerous papers in the area of Marketing Models, Optimization, Software Reliability and Inventory Management in international journals and conference proceedings. His current research interests include Marketing Models, Inventory Modeling, Software Reliability Growth Modeling, and Dynamic Optimization Techniques.

Dr. Alok Kumar is working as an Associate Professor in the area of Quantitative Techniques and Operations Management at FORE School of Management, New Delhi and are engaged in teaching the papers such as Business Statistics, Operations

Research, Business Research Methods, Supply Chain Management and Operations Management. He is a graduate in Mathematics (Hons.) from the University of Delhi and a post-graduate in Operational Research from Department of Operational Research, University of Delhi. He has been awarded Ph.D. degree in Operational Research in the area of Inventory Management at University of Delhi. His area of research interest is developing mathematical models in the field of inventory management and has published numerous research papers in refereed journals of national and international repute in the field of developing models for integration of innovation diffusion theory with inventory management. He also develops papers based on empirical research using research methodology techniques. He has several years of teaching and research experience. There are more than 25 research papers which have been published in international journals of high repute and that have been published by Springer, Taylor & Francis, Emerald, Inderscience etc., several research papers are published in conference proceedings, numerous research papers are published as book chapters and 11 research papers are published as working papers. Dr. Kumar has also conducted MDP in the area of decision making through quantitative techniques and FDP on Machine Learning & Data Analytics.

Dr. Naga Vamsi Krishna Jasti is Associate Professor in the Department of Mechanical Engineering at BITS Pilani. He is a PhD. From BITS Pilani and has 12 years of teaching and research experience. He has published around 30 research papers in reputed Scopus-indexed journals.