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Multi-Echelon Inventory Optimization under Disruption Risk

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ABSTRACT

This study investigates optimal inventory positioning in multi-stage supply chains by utilizing a mixed-integer nonlinear programming model that incorporates a scenario-based disruption delay, and the subsequent expediting to mitigate inventory shortages. The goal is to determine the amount and positioning of inventory in the supply chain to minimize the total inventory and expediting costs, subject to meeting customer service requirements. Discretization techniques are applied to linearize non-convex objective functions and reformulate the problem as an equivalent mixed-integer linear programming, which can be solved by the standard solvers. To demonstrate the model, numerical experiments are conducted to determine the optimal solution under different disruption scenarios. These experiments provide interesting insights regarding inventory decisions that can serve as guidelines for building resilience in the supply chain.

Keyword: inventory management, safety stock positioning, supply chain disruption, supply chain risk management

1. INTRODUCTION

Supply chain disruptions are caused by many natural and man-made events. The effects of these disruptions range from minor problems to bankruptcy and company closures (Shih 2020; Laato *et al*. 2020). As offshoring and globalization of manufacturing operations has grown in recent years, supply chains have become longer, more complex and geographically more diverse. Therefore, companies are more exposed to a variety of risks that can disrupt their operations (Simchi-Levi 2010, Wagner *et al*. 2014, Bode and Macdonald 2017, García-Arca *et al*. 2020, Liu *et al*. 2020, Ivanov and Dolgui 2020, Baghersad and Zobel 2021, Dolgui and Ivanov 2021).

Holding redundant inventory is one strategy to mitigate disruptions (Snyder *et al*. 2016, Pereira *et al*. 2020, Dolgui and Ivanov 2021, Taleizadeh *et al*. 2021). This strategy is often referred to as a just-in-case strategy, as opposed to a just-in-time strategy (Sheffi 2007). For example, in March 2011 a tsunami struck Japan resulting in the collapse of the Toyota's supply chain. After the tsunami, most of Toyota's Japanese plants were closed for nearly two months. In addition, Toyota's North American production was cut down to 30% percent in the subsequent six (6) months due to a shortage of 150 different parts to be produced in Toyota's Japanese plants (Canis 2011). Toyota experienced a 77% drop in profits in the second quarter of 2011, equivalent to \$1.36BN (MacKenzie *et al*. 2014). To be able to mitigate risks in case of future events of this type and magnitude, many Japanese automakers, including Toyota, started to hold a few months' worth of redundant inventory within their supply chains (Yoon *et al*. 2020, Li *et al*. 2021). However, as with other risk mitigation strategies, holding redundant inventory is costly. Digressing from just-in-time to just-in-case inventory management might be detrimental to product quality and to lean operations in general (Sheffi and Rice 2005, Zhang *et al*. 2018). Therefore, companies must carefully balance the trade-off between mitigation costs and supply chain disruption risks.

In a multi-echelon supply chain, the key questions are *where* and *how much* inventory are needed to meet customer requirements in normal times and during disruptions. However, these decisions are complex due to multiple factors, including the location and the probability of disruptions, delays during disruptions, the inventory unit cost at each stage and the cost of the post-disruption action. This makes it difficult to properly answer the where and how much inventory is needed questions and therefore, the best mitigation strategy. Given these difficulties, many companies tend to make sub-optimal decisions that result in excess inventories at several stages of the supply chain, holding inventories in the wrong locations or experiencing stock-outs and losing customers (Marcucci *et al*. 2021).

This paper supports decision-making in determining the right amount and the correct positioning of safety stocks to protect a serial supply chain from delay disruption. The decision variables are the lead times between stages because they form the basis for setting inventory targets. The inventory target for each stage is calculated by the amount that covers the net lead time. Net lead time is defined by subtracting the outbound lead time from the sum of the inbound lead time and processing time. In other words, one lead time affects the inventory target of the previous and the next processes. This is determined from the perspective of global optimality. In each stage, a delay occurs in the lead time with a probability for this delay (the disruption). If the inventory is insufficient dudelay; delay, expediting is performed as a post-disruption action to meet the service commitment. Unlike some of the previous work, the proposed model does not allow for backorders. This is consistent with a risk management perspective that considers measures in advance according to each scenario and establishes a system that can continue supply rather than accept the risk, and thus backorder (which assumes units will be available at a later time, after the delay is resolved). For some industries such as medical devices and pharmaceuticals, it would not be acceptable to "plan for" shortages and therefore, mitigation plans should be in place to continue supply even if extra costs are incurred. In such situations, the expediting approach considered in this study is more suitable.

The objective function is to minimize the expected cost, which is the sum of the inventory and the expediting costs. Since inventory cost is expressed as a concave function to express inventory pooling, this problem is a non-convex optimization problem. Discretization techniques are applied to linearize non-convex objective functions and reformulate the problem as mixed-integer linear programming, which can be solved by the standard solvers.

Numerical experiments were conducted in a 4-stage serial supply chain. In these experiments two types of supply chains that are characterized by the value added at each stage are compared. The experiments include several sensitivity analyses that consider factors related to the disruption probability, the location of the disruption and the cost structure. The following questions are addressed:

- How does the position and quantity of inventory held differ depending on the added value of each stage?
- How does the total cost differ depending on where the delay caused by the disruption occurs?
- Under what conditions should we stock and expedite in order to mitigate the delay?

The remainder of this paper is organized as follows. In Section 2, we review inventory mitigation literature for both single-echelon, and multi-echelon supply chains and discuss the contributions of this study. In Section 3, we present the proposed model to optimize inventory quantity and positioning in order to minimize the expected inventory holding and expediting costs under the presence of scenario-based delays. In Section 4, we present a thorough sensitivity analysis that considers multiple relevant factors. Finally, in Section 5 we present the results of the study and discuss future research directions.

2. LITERATURE REVIEW

In the past decade, academics and practitioners have become increasingly interested in the impact of supply chain disruptions, and relevant literature on the topic has increased significantly (Snyder *et al*. 2016, Xu *et al*. 2020). There are several review papers on disruption management, with, Snyder *et al*. (2016), Xu *et al*. (2020), Dolgui and Ivanov (2021) being three of the most recent works. In this study, we specifically review papers on inventory management under disruption. Section 2.1 discusses the single-echelon model, Section 2.2 describes the multi-echelon model, while Section 2.3 presents the contributions of this research.

2.1 Single-echelon Inventory Models under Disruption

Several papers study the optimal replenishment policy under disruption for the single-echelon inventory system, which determines when, from whom and how much to order. One research stream is to incorporate these disruptions into the economic order quantity (EOQ) model. Parlar and Berkin (1991) incorporated disruption into the EOQ model. In this study, the problem of determining the optimal order quantity during normal times and disruptions at random time intervals, is modeled. This model is referred to as the EOQD model. Berk and Arreola-Risa (1994) modified the model developed by Parallel and Berkin (1991) to derive a memoryless model in which normal and disrupted periods follow an exponential distribution. Weiss and Rosenthal (1992) developed an optimal inventory policy for EOQ inventory systems which may have a disruption in either supply or demand. Bar-Lev *et al*. (1993) extended the EOQD model under the assumption that the inventory level process is a Brownian motion with negative drift. Parlar and Perry (1996), and Gurler and Parlar (1997) extended the model into a multi-supplier setting. Parlar (2000) was able to provide a complete description of the cycle-related random variables for a stochastic inventory problem with supply interruptions. Ross *et al*. (2008) considered a dynamic environment: the probability of a disruption, as well as the demand intensity, can be time-dependent. Qi *et al*. (2009) extended the EOQD model to include disruptions at the supplier and at the firm itself. Snyder (2014) proposed a tight approximation for a continuous review inventory model with supplier disruptions. Bakal *et al*. (2017) proposed a model that allows buyers to place a disruption order based on disruption information. In a numerical experiment, they compared the case where there was no disruption order and discussed the value of disruption information. Konstantaras *et al*. (2019) studied the periodic review base stock (S, T) policy, which has an identical structure of EOQ with backorders.

Another modeling approach is the stochastic inventory system with disruption. Parlar *et al*. (1995) analyzed a (Q, r, T) inventory policy with deterministic and random yields when future supply is uncertain. Gupta (1996) studied the impact on operating costs of having an unreliable supplier in a continuous review (Q, r) policy. Song and Zipkin (1996) presented an inventory control model where the replenishment lead time changes over time. They proposed that the optimal policy has the same structure as in standard models, but its parameters change dynamically to reflect current supply conditions. They show that a longer lead time does not necessarily imply more inventory. Kalpakam and Sapna (1997) analyzed an environment-dependent (s, S) inventory system with renewal demands and lost sales where the environment goes through available and unavailable periods according to a two-state Markov chain. Moinzadeh and Aggarwal (1997) studied an unreliable bottleneck production/inventory system with a constant production and demand rate that is subject to random disruptions, and proposed an (s, S) production policy for such systems. Parlar (1997) proposed an ordering policy where the standard (q, r) policy is used when the supplier is available (ON), and as soon as the supplier is recovered, one orders enough to bring the inventory position up to the target level, when the supplier is unavailable (OFF). Arreola-Risa and DeCroix

(1998) proposed a modified (s, S) policy that if the inventory level is at or below s and the supply is available, an order is placed to bring the inventory level up to S. Ozekici and Parlar (1999) considered infinite-horizon periodic review inventory models with unreliable suppliers where the demand, the supply, and the cost parameters change with respect to a randomly changing environment. They show that a two-parameter, environment-dependent (s, S) policy is optimal. Gullu *et al*. (1999) analyzed a periodic review of the single-item inventory model under supply uncertainty with the objective of minimizing expected holding and backorder costs over a finite planning horizon under the supply constraints. Li *et al*. (2004) considered a periodic review inventory system subject to random demand and unreliable supply. The availability of supply is modeled as an alternating renewal process with general distributions for the duration of the UP and DOWN cycles. Mohebbi and Hao (2008) studied an unreliable supplier in a single-item stochastic inventory system that alternates randomly between two possible states (i.e., available and unavailable) following a two-state continuous time homogeneous Markov chain. Lewis *et al*. (2013) studied inventory management in global supply chains facing port-of-entry disruption risks and an infinite-horizon periodic review inventory control model is developed to determine the optimal average cost ordering policies under linear ordering costs with backlogged demand. Saithong and Lekhavat (2020) studied an optimal base-stock level with disruption and partial backlogging. They derived a closed-form expression of the optimal base-stock level that depends on the disruption duration. Taleizadeh *et al*. (2021) analyzed the effects of supply disruptions in a base stock (S, T) periodic review policy, in which the effect of disruptions on optimal review interval are considered, and supply disruption length is modeled as a discrete variable.

There are several single-echelon and finite-period approaches that consider inventory decisions. Kamalahmadi and Parast (2017) compared three mitigation strategies: pre-positioning inventory, backup suppliers and protected suppliers. Considering supply and environmental risks, they developed a two-stage mixed-integer programming model by utilizing the decision tree approach. Lucker *et al*. (2019) studied the integrated decision making of inventory and reserve capacity. They compared four strategies: inventory strategy, reserve capacity strategy, mixed strategy and passive acceptance. They clarified the selection conditions for each strategy according to product and supply chain characteristics. Karu and Singh (2020) studied disaster resilient proactive and reactive procurement models for humanitarian supply chains. They proposed a three-phase model. In the first phase, the resilient scores of suppliers are evaluated using the integrated DEMATEL and fuzzy-TOPSIS approach. In the second phase, the proactive model for the disaster resilient procurement is solved, in which the optimal procurement portfolio and inventory are determined. In the third phase, the reactive model is solved after the disruption is occurred.

2.2 Multi-echelon Inventory Models under Disruption

Several papers have studied multi-echelon inventory models with disruption. Kull and Closs (2008) demonstrated the interaction of inventory and supply risk on system performance for a two-echelon supply chain with a second-tier supply failure. Simulation modeling was used to understand the interactive impact of factors within given scenarios, demonstrating that increased inventory levels do not necessarily reduce the overall supply chain risks. Wu *et al*. (2010) proposed a performance evaluation model for a serial multi-echelon inventory system that considers backordering and lost sales. They proposed a Marko-chain modeling to evaluate the exact expected cost for a single-stage model and expand it to a multi-stage model with the approximated expected cost. Schmitt (2011) studied the best mix of multiple strategies to protect a supply chain from a disruption. The strategies include satisfying demand from an alternate location in the network, procuring material or transportation from an alternate source or route, and holding strategic inventory reserves throughout the network. They modeled the expected cost involving an inventory holding cost, a fixed cost to implement an option and a variable cost to reduce the response time, while satisfying service level commitments. DeCroix (2013) considered an assembly system with a single end product and a general assembly structure, where one or more of the component suppliers or (sub)assembly production processes is subject to random supply disruptions. They showed that a base-stock policy is not optimal.

He *et al*. (2013) introduced an alternate inventory policy and a heuristic to optimize its parameters. Computational tests suggest that their policy outperforms that of DeCroix (2013). He and Snyder (2013) considered a general distribution system (not restricted to two echelons) under disruption risk. Pal *et al*. (2014) studied a three-stage production-inventory system of suppliers, manufacturers and retailers in situations where disruptions occur. They studied optimal order batching decision-making in situations where disruption and machine breakdown occur at the same time. Yildiz *et al*. (2016) proposed a network design that considers the two objectives of cost minimization and reliability maximization. The Genetic Algorithm was applied to overcome the non-linearity of the objective function with respect to the reliability. Schmitt *et al*. (2017) proposed a model that considers pre-positioned inventory as a pre-disruption action and expediting as a post-disruption action. A simulation model was applied to the four-echelon supply chain. Swaik (2018) proposed a supply chain portfolio approach. In addition to scheduling, distribution and demand portfolio, pre-positioned inventory decisions were also considered in this two-stage stochastic programming model. Swaik (2019) compared the multi-period model with the two-period model and showed that the multi-period model allows for more detailed planning, as well as being more accurate in decision-making. Sawik (2020) proposed a two-period model for the multi-echelon supply chain that optimizes pre-disruption and post-disruption decision-making. He formulated a stochastic programming model, which determines inventory as a pre-disruption decision, and recovery and transshipment portfolio as a post-disruption decision.

He *et al*. (2019) integrated the decision-making of pricing and inventory in supply chain disruption under the assumption where there is a correlation between price and demand. They applied a real option approach and derived an optimal policy. De and Mahata (2019) studied EPQ in a three-stage supply chain with incomplete material quality, partial backordering and disruptions. They applied the Triangular dense fuzzy lock set approach. Yoon (2020) studied inventory and procurement decisions for the three-echelon supply chain composed of a first-tier supplier, a second-tier supplier and a manufacturer. They studied the value of sharing the disruption information of second-tier suppliers with manufacturers. Nguyen *et al*. (2020) studied the recovery scheduling of Multi-Echelon Assembly Supply Chain (MEASC) networks. They considered two recovery metrics: (i) minimizing the maximum tardiness of any order of the final product of the MEASC network and (ii) minimizing the time to recover from a disruptive event. They proposed decision rules that are applied locally at each manufacturer and are proven to optimize the two metrics.

2.3 Contributions of This Work

The proposed model is closely related to DeCroix (2013), He *et al*. (2013), and He and Snyder (2013). These models consider disruption risk and derive near-optimal parameters under the assumption of a particular ordering policy. These studies assume back-ordering when inventory is insufficient at each stage. Therefore, the shortage of inventory in one stage affects the other stages and the whole system becomes a complicated stochastic system. The model proposed in this research is different from these studies as it assumes that expediting will completely cover the inventory shortage and, therefore, the inventory shortage in one stage will not affect the other stages. While in practice it is impossible to guarantee that expediting will always meet the requirements from a disruption, the strategic planning of expediting to mitigate shortages should result on a high certainty level if resources are prepositioned to expedite and the required technology in place to handle the disruption. This research makes three contributions. First, the relationship between disruptions and inventory planning strategies, which has mostly been discussed at a single stage, is extended into multiple stages. Second, it provides a different and richer interpretation of the multiple stage framework and its applicability to practice based on the analysis completed in the numerical experiments. Third, this paper analyzes the trade-off between whether to have protection inventory as a pre-disruption strategy or to expedite as a post-disruption strategy. Unlike previous work, the proposed model does not allow backordering as a mitigation strategy, a view that represents systems where it is not acceptable to plan for shortages. In other words, the plan must be to always meet the demand (service guarantee) based on protection inventory or by expedited production/inventory.

3. MODELING FRAMEWORK

The following section describes the modeling framework based on the GS model. Section 3.1 presents a summary of the notation. Section 3.2 presents the modeling framework. In Section 3.3, we present the formulation, while Section 3.4 presents the discretization.

3.1 Notation

The summary of the notation used is presented as follows:

Set

- $N = \{i = 1, \cdots, n\}$: A set of stages
- $S = \{k = 1, \cdots, m\}$: A set of scenarios

Parameter

- μ: mean demand at stage n
- σ: standard deviation of demand at stage n
- s_n^{out}: required service time from external customer at stage n
- s_1^{in}: procurement lead time from external supplier at stage 1
- p_{ik}: processing time at stage i in scenario k
- π_k: probability of scenario k $(k = 1, \cdots, m)$
- h_i: unit inventory holding cost at stage i $(i = 1, \cdots, n)$
- e_i: expedited inventory cost at stage i $(i = 1, \cdots, n)$
- z: safety factor
- r: the number of discrete supports

Decision Variables

- s_i^{in}: inbound service time at stage i $(i = 2, \cdots, n)$
- s_i^{out}: outbound service time at stage i $(i = 1, \cdots, n - 1)$
- L_i^{plan}: the planned net replenishment time at stage i $(i = 1, \cdots, n)$
- L_{ik}^{real}: the realized net replenishment time at stage i in scenario k $(i = 1, \cdots, n;\ k = 1, \cdots, m)$
- x_{ir}: binary variable to take 1, if L_i^{plan} take the discrete support value r and 0 otherwise.

3.2 Assumptions

Network

A supply chain is modeled as a multi-stage serial network with n stages. Let $N = \{i = 1, \cdots, n\}$ be a set of stages, as illustrated in **Figure 1**. Each stage represents a point where inventory in the supply chain is held. The model assumes the supply of a single item, where for example stage 1 represents the inventory of the key raw material and stage n is the point where inventory is shipped to the customer. Stage i receives replenishment from stage $i - 1$, for $i = 2$ to n.

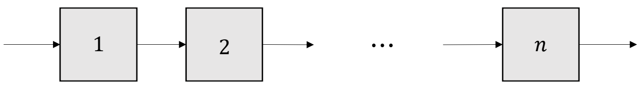

Figure 1. A serial supply chain

Lead time

The proposed model determines the tactical placement of inventory to be used in the event of a disruption. **Figure 2** describes the model's conceptual basis. Since the time to recover from the disruption is unknown, we assume steady-state conditions for an infinite period. The model assumes there is a set of possible scenarios called S, where the first scenario represents the baseline condition (no disruptions), and scenarios $2\ to\ m$ represent disruption states; there is a total of m scenarios and $S = \{k = 1, \cdots, m\}$. For each disruption scenario the production capacity of one or more stages of the supply chain is degraded, causing production delays. Let p_{ik} be the realized processing time of stage i in scenario k.

Figure 2. The relationship of decision, risk event, and disruption

Each stage quotes the outbound service time s_i^{out} for the downstream stage. This becomes the inbound service time s_{i+1}^{in} for the downstream stage $i+1$ (*i.e.*, $s_i^{out} = s_{i+1}^{in}$). **Figure 3** illustrates the relationship between s_i^{out}, s_i^{in} and p_{ik}. Let L_i^{plan} denote the planned net lead time of stage i with no disruption and let L_{ik}^{real} denote the realized net lead time of stage i in scenario k. L_{ik}^{real} is defined as in the equation (1).

$$L_{ik}^{real} = s_i^{in} + p_{ik} - s_i^{out} \tag{1}$$

The realized processing time p_{ik}, the required lead time s_n^{out} from the external customer and the procurement lead time s_1^{in} from the external supplier are given parameters. Inbound service time s_i^{in} for stages $i = 2, \cdots, n$, outbound service time s_i^{out} for stages $i = 1, \cdots, n-1$, the net lead time before and after interruption L_i^{plan}, L_{ik}^{real} are the decision variables.

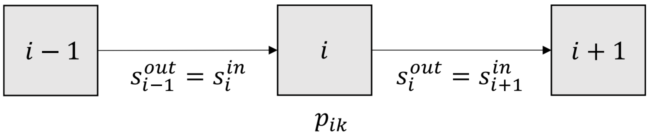

Figure 3. The relationship between s_i^{out}, s_i^{in} and p_{ik} for each stage k

Planned Inventory Level

The demand for items consumed in stage n per period is assumed to follow a normal distribution with mean, μ, and standard deviation, σ. In this study, a period represents a standard time segment to manage inventory and usually represents one day (or one week, or one month).

The planned inventory level at stage i is PI_i. This is called the planned inventory because it covers the demand during the net lead time L_i^{plan} assumed at the time of planning and is calculated according to equation (2):

$$PI_i = \mu L_i^{plan} + z\sigma\sqrt{L_i^{plan}} \tag{2}$$

where the first term is the average inventory of the planned net lead time period of stage i and the second term is the safety stock of the planned net lead time period of stage i.

The inventory levels of stage $1, \cdots, n-1$ shown in equation (2) are based on the average (μ) and standard deviation (σ) of the demand at the final stage n. While the per period average and standard deviation of the demand at stage $i = 1, \cdots, n-1$ would depend on the ordering policy of the downstream stage $i+1$, the use of the characterization from stage n for all the preceding stages is in line with today's information sharing capabilities and with the scope of the model (tactical level planning based on steady-state conditions).

Stage cost for each scenario

The average realized demand, RD_{ik}, during the realized lead time period, L_{ik}^{real}, is calculated as shown in equation (3):

$$RD_{ik} = \mu L_{ik}^{real} \tag{3}$$

The inventory level in scenario k can be divided into the following cases depending on the availability of inventory to meet the realized demand.

Case 1: $PI_i \geq RD_{ik}$

In this case, the planned inventory is sufficient to cover the realized demand. The expected inventory level in stage i is assumed to be $(I_i^{max} + I_i^{min})/2$ as an approximate expected value, where I_i^{max} and I_i^{min} are the maximum and minimum inventory levels at stage i. Substituting $I_i^{max} = PI_i$, $I_i^{min} = PI_i - RD_{ik}$ results in an expected inventory level of $(2PI_i - RD_{ik})/2$. By multiplying this approximate expected inventory level by the inventory unit cost h_i of stage i, the inventory cost c_{ik} in scenario k of stage i can be derived as shown in equation (4):

$$c_{ik} = \frac{h_i}{2}(2PI_i - RD_{ik}) \tag{4}$$

Case 2: $PI_i < RD_{ik}$

In this case, the planned inventory is not sufficient to cover the realized demand, therefore it would be fully consumed during the period. The expected inventory level in stage i becomes $PI_i/2$ by substituting $I_i^{max} = PI_i$, $I_i^{min} = 0$. In addition, it is assumed that the difference between the realized demand and the planned inventory will be covered by expedited units rather than backordering. The expediting assumption includes overtime production, special delivery and outsourcing. Assuming that e_i is the expediting cost per unit, the cost of scenario k in stage i in Case 2 can be derived as shown in equation (5):

$$c_{ik} = \frac{h_i}{2}PI_i + e_i(RD_{ik} - PI_i) \tag{5}$$

Note that it is expected that the cost of expedited production/inventory at stage i, (e_i) would be significantly larger than the holding costs at stage i, (h_i), as this cost represents substantial efforts to produce/deliver the units during a disruption.

In the optimization model, the objective function is to minimize the cost for each scenario defined as in equation (6) by taking the smaller of equations (4) and (5), since either case 1 or case 2 is active.

$$c_{ik} = \min\left(\frac{h_i}{2}(2PI_i - RA_{ik}), \frac{h_i}{2}PI_i + e_i(RA_{ik} - 2PI_i)\right) \tag{6}$$

3.3 *Formulation*

The model for determining the pre-positioned inventory of the supply chain is shown as in (7a) to (7i):

$$\text{minimize} \quad \sum_{i=1}^{n}\sum_{k=1}^{m}\pi_k c_{ik} \tag{7a}$$

$$\text{subject to} \quad c_{ik} \geq \frac{h_i}{2}(PI_i - RA_{ik}), \quad i = 1, \cdots, n,\ k = 1, \cdots, m \tag{7b}$$

$$c_{ik} \geq \frac{h_i}{2}PI_i + e_i(RA_{ik} - PI_i), \quad i = 1, \cdots, n,\ k = 1, \cdots, m \tag{7c}$$

$$PI_i = \mu_i L_i^{plan} + z\sqrt{L_i^{plan}}\sigma_i, \quad i = 1, \cdots, n \tag{7d}$$

$$RA_{ik} = \mu_i L_{ik}^{real}, \quad i = 1, \cdots, n,\ k = 1, \cdots, m \tag{7e}$$

$$L_{ik}^{real} = s_i^{in} + p_{ik} - s_i^{out}, \quad i = 1, \cdots, n,\ k = 1, \cdots, m \tag{7f}$$

$$s_i^{out} = s_{i+1}^{in}, \quad i = 1, \cdots, n-1 \tag{7g}$$

$$s_i^{out}, s_i^{in}, L_i^{plan} \in Z^+, \quad i = 1, \cdots, n \tag{7h}$$

$$L_{ik}^{real} \in Z^+. \quad i = 1, \cdots, n,\ k = 1, \cdots, m \tag{7i}$$

The objective function (7a) is the minimization of the approximated expected value of the cost under all the scenarios after the disruption. Constraints (7b) and (7c) define the cost of each scenario. If $PI_i > RA_{ik}$, then constraint (7b) is active, otherwise constraint (7c) is active. Constraints (7d) define the value of PI_i, while constraints (7e) define RA_{ik}. Constraints (7f) determine the value of the net lead time L_{ik}^{real}. Constraints (7g) show that the outbound service time of upstream stage equals the inbound service time of the downstream stage. Constraints (7h) define the decision variables $s_i^{out}, s_i^{in}, L_i^{plan}$ are non-negative integers, while constraints (7i) ensure the non-negativity of L_{ik}^{real}. Note that L_{ik}^{real} takes only non-negative integer value because of constraints (7f) and that in the implementation of the model parameters: $s_i^{in}, p_{ik},\ s_i^{out}$ are only considered as integer values.

3.4 Discretization

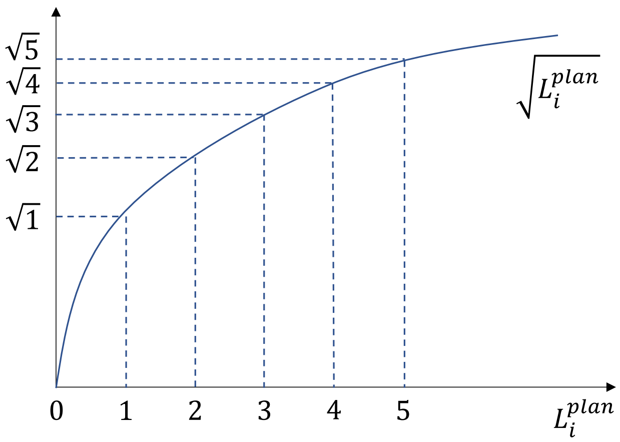

Figure 4. Discretization of concave cost function

As the definition of safety stock in (7d) uses a square root function, the model is a concave minimization problem and can be reformulated as an equivalent Mixed Integer Linear Programming (MILP) by using the discretization. **Figure 4** illustrates the discretization of concave cost function. We consider a nonlinear problem to minimize the concave function as a building block, as shown in equation (8):

$$\begin{aligned} \text{minimize} \quad & \sqrt{L_i^{plan}} \\ \text{subject to} \quad & L_i^{plan} \geq 0 \end{aligned} \tag{8}$$

Let $r = 0, \cdots, R$ denote a set of the discrete supports in the function $\sqrt{L_i^{plan}}$. Let x_{ir} denote the binary variable to take one if $L_i^{plan} = r$ is selected and 0, otherwise. The nonlinear optimization problem (8) can be approximated via an integer linear programming problem as in (9a) to (9d):

$$\begin{aligned} \text{minimize} \quad & \textstyle\sum_{r=0}^{R} \sqrt{r} x_{ir} && (9a) \\ \text{subject to} \quad & \textstyle\sum_{r=0}^{R} x_{ir} = 1, && (9b) \\ & \textstyle\sum_{r=0}^{R} r x_{ir} = L_i^{plan} && (9c) \\ & x_{ir} \in \{0,1\}, r = \{0, \cdots, R\} && (9d) \end{aligned}$$

The objective function (9a) is the sum of the discrete supports. Constraint (9b) requires that only one discrete support among $r = 0, \cdots, R$ is selected. Constraint (9c) requires that the L_i^{plan} takes the value r, if x_{ir} is selected. Constraint (9d) requires a binary condition of x_{ir}.

Using this representation, the problem can be modeled via MILP as in (10a) to (10l):

$$\begin{aligned} \text{minimize} \quad & \textstyle\sum_{i=1}^{n} \sum_{k=1}^{K} \pi_k c_{ik} && (10a) \\ \text{subject to} \quad & c_{ik} \geq \frac{h_i}{2}(PI_i - RA_{ik}), \\ & i = 1, \cdots, n;\ k = 1, \cdots, K && (10b) \\ & c_{ik} \geq \frac{h_i}{2} PI_i + e_i(RA_{ik} - PI_i), \\ & i = 1, \cdots, n;\ k = 1, \cdots, K && (10c) \\ & PI_i = \mu_i L_i^{plan} + z \textstyle\sum_{r=0}^{R} \sqrt{r} x_{ir} \sigma_i, \\ & i = 1, \cdots, n && (10d) \\ & \textstyle\sum_{r=0}^{R} x_{ir} = 1, \quad i = 1, \cdots, n && (10e) \\ & \textstyle\sum_{r=0}^{R} r x_{ir} = L_i^{plan} \quad i = 1, \cdots, n && (10f) \\ & RA_{ik} = \mu_i L_{ik}^{real}, \\ & i = 1, \cdots, n;\ k = 1, \cdots, K && (10g) \\ & L_{ik}^{real} = s_i^{in} + p_{ik} - s_i^{out}, \\ & i = 1, \cdots, n;\ k = 1, \cdots, K && (10h) \\ & s_i^{out} = s_{i+1}^{in}, \quad i = 1, \cdots, n-1 && (10i) \\ & x_{ir} \in \{0,1\}, \quad r = 0, \cdots, R && (10j) \\ & s_i^{out}, s_i^{in}, L_i^{plan} \in Z^+,\ i = 1, \cdots, n && (10k) \\ & L_{ik}^{real} \geq 0.\ i = 1, \cdots, n;\ k = 1, \cdots, K && (10l) \end{aligned}$$

Thus, the problem transforms into a MILP model that can be solved via standard off-the-shelf MILP solvers.

4. MODEL EXAMPLE

This section presents an example to illustrate how the proposed model supports inventory decision in a multi-echelon supply chain under disruption risk. The example consists of a four-echelon serial supply chain, as illustrated in **Figure 5**, where the first stage relates to a supplier that provides the key raw material, the second stage represents a factory that processes the material, the third stage represents an assembly operation that adds other components and finalizes the product and the fourth stage represents a distribution process that delivers to the end customers. The model assumes there are two possible scenarios: normal operation and disrupted operation, therefore $m = 2$ and $K = \{1(normal), 2(disruption)\}$. In the disrupted scenario at least one of the stages will undergo a production delay.

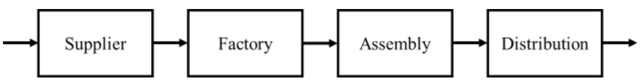

Figure 5. Four-echelon serial supply chain

4.1 Baseline Problem Description with No Disruptions

Demand follows a normal distribution with $\mu_i = 100, \sigma_i = 10$ for $\forall i \in N$, which represents a system with low demand variability. The inbound lead time from the external supplier (tier 2 supplier) is $s_1^{in} = 0$, and the outbound lead time to external customer is $s_4 = 0$. The safety stock parameter (z) is set to 1.96 (95% confidence level), thus representing a system with a relatively high level of customer service. The processing time at each stage is one time unit.

Two baseline cases are considered. In baseline case 1, the inventory holding cost is \$25 for stage 1 and increases by another \$25 for each subsequent stage (thus, $i_4 = \$100$). In baseline case 2, the inventory holding cost for stage 1 is \$70, and increases by \$10 for each subsequent stage (thus, i_4 is also \$100). The first case serves to represent a supply chain where there is a significant value adding process as the item moves towards the customer (flows from one stage to the next), while the second case represents a case where there is less value addition as it moves towards the customer (raw material costs represent a larger percentage of the end item's cost). The first case is called HSC: High incremental value Supply Chain, while the second case is called LSC: Low incremental value Supply Chain.

The results for the baseline cases with no disruption is given in **Table 1**, where columns 2 to 5 indicate the optimal inventory levels and the last column indicates the resulting total costs. In the HSC baseline case, each stage gets one time unit worth of inventory, while in the LSC baseline case inventory is placed in stages 2 and 4. This illustrates the relationship between the incremental value of the supply chain (how much the value increases per stage) and the optimal inventory positions, even with no disruptions. Clearly, the total costs for the HSC baseline are lower than those for the LSC baseline case as the inventory cost is lower in the first three stages.

Table 1. Baseline case results

	Optimal Inventory Levels				Total Costs
	τ_1	τ_2	τ_3	τ_4	
HSC	1	1	1	1	17,400
LSC	0	2	0	2	22,989

4.2 Disruption Cases

This section analyzes the optimal inventory positioning decisions when there is the possibility of disruptions. Seven disruption cases are analyzed, each with different disruption scenarios (the particular stages that "fail"). The seven cases are illustrated in **Figure 6**, where the stage in red color indicates a disruption at that point in the supply chain. The first four cases represent a disruption in a single stage of the chain. The fifth case represents a scenario where both production stages have a disruption and the sixth case where all the internal stages (assuming distribution is internal) are disrupted, while the last case is where all the stages suffer a disruption. It is proposed that the simultaneous disruption of multiple stages would typically relate to natural events that affect a region, for example a major storm, an earthquake or a tsunami.

It is noted that additional cases could be formed, but this set of conditions would allow demonstrating the applicability of the model and the sensitivity of the inventory decisions to the business parameters. The delay associated with a disruption at a stage is four time units (per stage with a disruption) and the probability of the disruption is 10% ($\pi_{2(disruption)} = 10\%$). The expedited inventory cost is based on the expediting to holding cost ratio experimental parameter α, where the expedited inventory cost is the multiplication of α by the inventory holding cost ($e_i = \alpha \times h_i$). The value is set to 10 ($\alpha = 10$), thus for example, for HSC, $e_1 = \$250$ and for LSC, $e_1 = \$700$, while for both cases, $e_4 = \$1{,}000$.

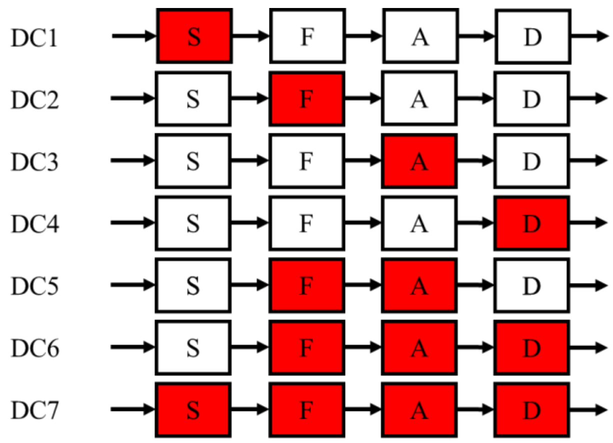

Figure 6. Set of disruption cases under analysis

Table 2. Inventory decisions and metrics with $\alpha = 10$ *and* $\pi_{2(disruption)} = 10\%$

	HSC								LSC						
Case	τ_1	τ_2	τ_3	τ_4	TI	%E	C*		τ_1	τ_2	τ_3	τ_4	TI	%E	C*
Baseline	1	1	1	1	4	0%	1.00		0	2	0	2	4	0%	1.00
DC1	3	1	1	1	6	45%	1.39		3	1	0	2	6	61%	1.83
DC2	1	3	1	1	6	60%	1.77		0	4	0	2	6	62%	1.91
DC3	1	1	3	1	6	67%	2.16		0	0	5	1	6	62%	2.02
DC4	1	1	0	4	6	69%	2.53		1	0	0	5	6	64%	2.12
DC5	1	3	3	1	8	74%	2.93		0	4	3	1	8	73%	2.98
DC6	1	3	3	3	10	79%	4.47		0	4	2	3	9	86%	4.17
DC7	3	3	3	3	12	80%	4.86		3	3	3	3	12	80%	5.00

The model's results for the baseline and the seven disruption cases for the HSC and the LSC supply chain types are presented in **Table 2**. The table presents the optimal inventory decision per stage (τ_1), followed by the total system inventory ($\text{TI} = \tau_1 + \tau_2 + \tau_3 + \tau_4$). The next column provides the percentage of costs represented by expediting (%E) under the disruption scenario, where a value of 0% would indicate no expediting costs are incurred during that disruption case as there is enough inventory to cover the disruption. Column C* indicates the cost ratio of each case versus the baseline (with no disruption in **Table 1**).

The optimal decisions for both the HSC and LSC involve placing protection inventory at the stage where the possible disruption has been identified in all seven disruption cases. For the four cases with a single stage disruption (DC1 to DC4), the protection inventory is one unit less than what would be required for "full protection" (TI would need to equal 7), therefore, the optimal decision includes some expediting when there is a disruption. For multi-stage failure cases (DC5 - DC7), the protection inventory is two or more units fewer than required for "full protection", with the largest difference observed in DC7, where full protection would be obtained by TI = 16, but the optimal solution is TI = 12, under both supply chain types. Case DC6 is one where the total inventory is different based on the type of supply chain (TI = 8 for HSC, and TI = 9 for LSC), where in LSC there would be a higher dependency on expedited units (less protection inventory). The results also show that the placement of the inventory depends on the type of supply chain. The placement of inventory in the HSC is "balanced", where there is inventory in each stage (with the exception of DC4, where stage 3 has 0 inventory). On the other hand, and in line with the results observed when there are no disruptions, inventory is placed in fewer stages for the LSC, where in six out of the seven cases at least one stage has no inventory. Only DC7 represents a case where the optimal placement of inventory is the same for HSC and LSC. In terms of the metrics, the costs associated with expediting, E%, and the ratio of costs versus the baseline C* increases as the disruption occurs in a stage of the process closer to the customer or as the number of disrupted stages increases. The difference between the supply chain types and model parameters is noted in the change in this value from DC1 to DC4. For the HSC the value of C* increased from 1.39 to 2.53 as the disruption "moved" to a later stage, while in the LSC the value of C* increased from 1.83 to 2.12. The overall increase from baseline to the worst disruption case is close to fivefold.

4.3 Sensitivity to the Probability of Disruption

A critical element associated with contingency planning is the probability of the disruption. As model results are sensitive to the parameters used, it is relevant to analyze the relationships between inventory decisions and the probability of disruption. This section first describes the effect of a lower probability of disruption on the inventory decisions where the probability of the disruption is changed to 5% ($\pi_{2(disruption)} = 5\%$). The results presented in **Table 3** indicate a reduction in the probability of the disruption has a very significant effect on the overall inventory decisions. For all the seven disruption cases there is no protection inventory regardless of the type of supply chain; all TI values equal 4. In other words, the optimal decision is to count solely on expedited inventory when there is a disruption, which can be expected given the low probability of the disruption. The cells shaded in grey represent changes in the decision between $\pi_{2(disruption)} = 10\%$ and $\pi_{2(disruption)} = 15\%$. For all seven disruption cases of the HSC, the optimal decision is to have one unit of inventory per stage, while for LSC, inventory is unevenly placed with the exception of the "all stages" disrupted case (DC7). Therefore, even when the probability of the disruption is low, inventory positioning decisions would depend on the type of supply chain. When considering the metrics, the %E is much higher than in the original environment because the optimal decision is to expedite when there is a disruption. The ratio of costs to the baseline increased as well as the disruption cases location and size, but at a slower rate than in the condition of $\pi_{2(disruption)} = 10\%$, about a three-fold increase from baseline to the worst disruption case, thus in general about 40% lower costs.

Table 3. Inventory decisions and metrics with $\alpha = 10, and\ \pi_{2(disruption)} = 5\%$

	HSC								LSC						
DC	τ_1	τ_2	τ_3	τ_4	IP	%E	C*		τ_1	τ_2	τ_3	τ_4	IP	%E	C*
None	1	1	1	1	4	0%	1.00		0	2	0	2	4	0%	1.00
DC1	1	1	1	1	4	80%	1.20		1	1	0	2	4	90%	1.44
DC2	1	1	1	1	4	89%	1.40		0	2	0	2	4	91%	1.47
DC3	1	1	1	1	4	93%	1.60		0	0	3	1	4	92%	1.54
DC4	1	1	1	1	4	94%	1.80		1	0	0	3	4	93%	1.59
DC5	1	1	1	1	4	96%	2.00		0	2	1	1	4	96%	2.04
DC6	1	1	1	1	4	98%	2.81		0	2	1	1	4	97%	2.65
DC7	1	1	1	1	4	98%	3.01		1	1	1	1	4	98%	3.10

As a second analysis, the probability of disruption is increased to 15% ($\pi_{2(disruption)} = 15\%$). **Table 4** presents the results where the cells in grey indicate a change from the condition where $\pi_{2(disruption)} = 10\%$. As can be observed in **Table 4**, the increase in the disruption probability did not change the optimal decisions for 6 of the 7 disruption cases under the HSC environment with the exception being case DC2. The optimal decision under DC2 is to have "full" protection inventory in the stage with a possible disruption, therefore, for DC2 under the HSC type, there is no planned expediting (%E = 0%). The optimal inventory decisions changed for 3 cases under the LSC (DC4, DC6, DC7) where in DC4 the location of the inventory changed (but not the total), while in DC6 and DC7, the overall quantities of protection inventory increased. Note that the C* at DC7 is 5.25 and 5.4 for the HSC and the LSC, respectively. This represents a relatively small increase versus the condition with $\pi_{2(disruption)} = 10\%$ (where C* = 4.86 and 5, respectively).

Table 4. Inventory decisions and metrics with $\alpha = 10, and\ \pi_{2(disruption)} = 15\%$

	HSC								LSC						
DC	τ_1	τ_2	τ_3	τ_4	IP	%E	C*		τ_1	τ_2	τ_3	τ_4	IP	%E	C*
None	1	1	1	1	4	0%	1.00		0	2	0	2	4	0%	1.00
DC1	3	1	1	1	6	45%	1.43		3	1	0	2	6	61%	1.91
DC2	1	4	1	1	7	0%	1.85		0	4	0	2	6	62%	1.99
DC3	1	1	3	1	6	67%	2.28		0	0	5	1	6	62%	2.10
DC4	1	1	0	4	6	69%	2.67		0	0	0	6	6	62%	2.21
DC5	1	3	3	1	8	74%	3.13		0	4	3	1	8	73%	3.17
DC6	1	3	3	3	10	79%	4.83		0	4	3	4	11	66%	4.45
DC7	3	3	3	3	12	80%	5.25		3	4	3	4	14	61%	5.40

While the full results are not presented, an analysis was performed with $\pi_{2(disruption)} = 20\%$. At this probability of disruption and for both types of supply chain, the protection inventory increased as to provide "full" coverage for all the disruption cases. For example, in DC7 the optimal decision is $\tau_1 = 4, \tau_2 = 4, \tau_3 = 4$ and $\tau_4 = 4$. In other words, the optimal plan when $\pi_{2(disruption)} = 20\%$ is never to expedite, and therefore for all disruption cases %E = 0%. While this is an expected result, it illustrates one benefit of the model, the ability to provide information that would allow decision makers to understand the relationship between the probability of disruption and different inventory configurations: no protection, partial protection, or full protection.

4.3 *Sensitivity to* Expediting Costs

Another relevant factor in the described environment is the cost associated with expediting. The initial analysis had the expediting to holding cost ratio equal to 10 ($\alpha = 10$). This value is first decreased by 60% ($\alpha = 4$), noting that $\pi_{2(disruption)}$ is returned to its initial value of 10% ($\pi_{2(disruption)} = 10\%$). The results are presented in **Table 5**, where the shaded cells indicate a change from the results in **Table 2** ($\alpha = 10, \pi_{2(disruption)} = 10\%$). The results are similar to those obtained when the probability of disruption was reduced to 5% (presented in **Table 3**), a "move" towards full dependency on expediting inventory when there is a disruption. However, the location of the inventory is not the same between the two situations for the LSC. For example, in this set of conditions the optimal solution DC7 is $\tau_1 = 1, \tau_2 = 1, \tau_3 = 0$ and $\tau_4 = 2$, while when the disruption probability is lower (**Table 3**), each stage had one unit of inventory. Given the plan is to expedite in the case of a disruption, the values of %E are higher than at the original condition. As in previous analysis, the value of C* increases as the disrupted stage moves towards the customer or where there are more stages with a disruption, but it is noted the values are lower by 30 to 40% from the condition with $\alpha = 10$.

Table 5. Inventory decisions and metrics with $\alpha = 4, and\ \pi_{2(disruption)} = 10\%$

	HSC								LSC						
DC	τ_1	τ_2	τ_3	τ_4	IP	%E	C*		τ_1	τ_2	τ_3	τ_4	IP	%E	C*
None	1	1	1	1	4	0%	1.00		0	2	0	2	4	0%	1.00
DC1	1	1	1	1	4	62%	1.16		1	1	0	2	4	78%	1.35
DC2	1	1	1	1	4	77%	1.32		0	2	0	2	4	80%	1.37
DC3	1	1	1	1	4	83%	1.48		0	0	3	1	4	81%	1.43
DC4	1	1	1	1	4	87%	1.64		1	0	0	3	4	83%	1.47
DC5	1	1	1	1	4	90%	1.80		0	2	1	1	4	90%	1.83
DC6	1	1	1	1	4	94%	2.44		0	2	0	2	4	94%	2.31
DC7	1	1	1	1	4	95%	2.60		1	1	0	2	4	95%	2.67

Table 6. Inventory decisions and metrics with $\alpha = 16$ $and\ \pi_{2(disruption)} = 10\%$

	HSC								LSC						
DC	τ_1	τ_2	τ_3	τ_4	IP	%E	C*		τ_1	τ_2	τ_3	τ_4	IP	%E	C*
None	1	1	1	1	4	0%	1.00		0	2	0	2	4	0%	1.00
DC1	4	1	1	1	7	0%	1.44		4	1	0	2	7	0%	1.94
DC2	1	4	1	1	7	0%	1.88		0	4	1	1	6	72%	2.05
DC3	1	1	4	1	7	0%	2.31		0	0	5	1	6	72%	2.15
DC4	1	1	0	4	6	78%	2.74		0	0	0	6	6	72%	2.26
DC5	1	4	4	1	10	0%	3.19		0	4	4	1	9	63%	3.24
DC6	1	4	4	4	13	0%	4.94		0	4	4	4	12	55%	4.57
DC7	4	4	4	4	16	0%	5.38		4	4	4	4	16	0%	5.53

To conclude the analysis, the value of α is increased by 60% ($\alpha = 16$) and the results are presented in **Table 6**. As expected, higher expediting costs would lead to an increase in protection inventory, where in the HSC there is full protection in six out of the seven disruption cases, the exception being DC4. The opposite occurs for the LSC, where there is full protection in only two disruption cases (DC1 and DC7). It is noted, that only one unit of inventory is unprotected in all cases with expediting. These confirm previous results that demonstrate the positioning of the inventory depends on the type of supply chain. Finally, the C* has a similar behavior as in the previous analysis noting

4.4 Sensitivity Summary Example: Case DC5

In order to better illustrate the analysis capabilities of the proposed model, a graphical summary of one of the disruption cases is presented in this subsection. Case DC5 with two disrupted stages was selected and a graphical sensitivity analysis is presented in **Figure 7**. The figure describes the combination of disruption probability and

that the costs obtained here are the highest, but this is expected with a higher α.

expediting to holding cost ratio parameters where the optimal decision would be a strategy that depends solely on expediting (the grey area), a strategy that combines protection inventory with expediting (blue and green areas), and a strategy solely based on protection inventory (yellow). Besides illustrating the different types of optimal strategies, the figure shows the differences in protection inventory amounts and locations based on these parameters and the type of supply chain.

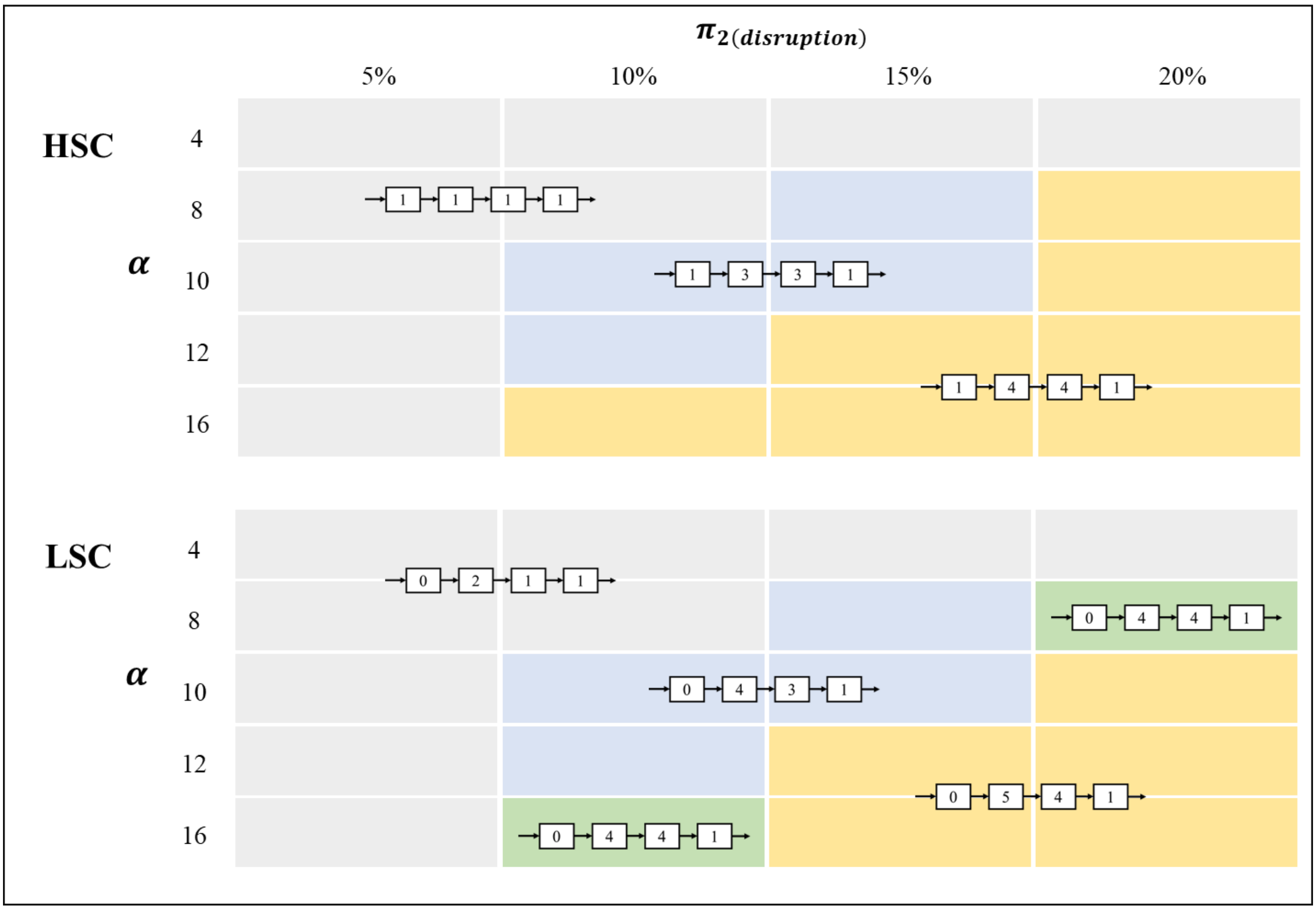

Figure 7. Sensitivity summary example

4.5 Managerial implications

The numerical experiments suggest the following managerial implications:

Implication 1: *The position where inventory should be held differs between the two supply chain types.*

For the majority of the cases the HSC and the LSC resulted in different inventory positioning. The inventory tended to be decentralized or "spread out" in the HSC type, while it was often centralized in the LSC type. In most scenarios, there is some inventory in each of the stages for the HSC type, while there are multiple scenarios where the planned inventory is 0 for one or more stages for the LSC type. It is argued that given the higher inventory costs of the LSC, there is a benefit to the risk pooling effect of "using" the centralized approach. On the other hand, when multiple disruption points are possible, it is often better to decentralize inventory depending on their location and the supply chain characteristics. This suggests that optimizing for a single stage is not sufficient when considering disruptions, demonstrating the importance and effectiveness of the global optimization strategy this study considers.

Implication 2: *The numbers and location of disruptions have a significant impact on costs.*

A supply chain that is susceptible to the simultaneous disruption of multiple stages due to physical proximity (location in the same "region") or codependency on critical systems / suppliers requires mitigation strategies with a significant price tag to be able to meet its commitment to customers. The proposed model helps characterize the costs of systems with single or multiple stage disruption, and thus analyzing the benefits of spreading out stages geographically to "eliminate" the possibility of simultaneous disruptions. However, this would lead to an increase in transportation and other "movement" related costs. This suggests that managers need to consider the trade-off between transportation costs and geographic dispersion that limits or eliminates simultaneous multi-stage disruptions.

Implication 3: *Combining inventory mitigation and expediting strategies reduces total costs.*

Inventory mitigation becomes effective when the probability of disruption is high or the cost of expediting is high. On the other hand, under most scenarios, full protection by inventory mitigation is rarely achieved. For this reason, it is not desirable to view inventory as the sole alternative and to consider countermeasures based on expediting such as capacity reserves and production flexibility. Thus, combining pre-disruption and post-disruption strategies as a mechanism to meet the system requirements reduces total costs and prevents the disruption to negatively impact the customers.

5 CONCLUSIONS

In today's complex global supply chains disruptions are unavoidable, thus effective mitigation strategies and contingency planning is very critical. Two typical methods to mitigate supply chain disruptions are holding protection inventory or having capabilities to expedite production / inventories at a premium cost. There are several critical questions around these strategies including how much protection inventory to hold and where to position it, which in turn dictates the level of dependency on expedited production / inventories. These questions have not been fully addressed by previous research in the case of multi-stage supply chains with service level commitments. This paper addresses this gap in literature.

In this paper, we proposed a model that incorporates scenario-based delay into the multi-echelon inventory model. For each scenario, the delay in processing time is an input. Expediting is performed to continue the service when the inventory is insufficient. The goal is to determine the amount and position of inventories in a supply chain, so as to minimize total inventory costs to guarantee the target demand quantity by the target service time with the delay under all the considered scenarios. In numerical experiments, the optimal solution was derived and compared under different disruption stages and probabilities. Three key managerial implications were reached: (1) the inventory positioning differs between the HSC and the LSC supply chain types; (2) the numbers and location of disruptions have a significant impact on costs; and (3) combining inventory mitigation and expediting reduces total costs.

Future work includes considering a model that incorporates network design and investment actions to reduce disruption probability. Another direction is to expand from the serial supply chain to the general network type supply chain where a stage might have multiple successors or predecessors. It is known that the network topology has a significant impact on supply chain risk. When a supply chain stage is serving many downstream stages and the demand for each store is an independent random variable, holding inventory at an upstream stage has an advantage in terms of decreasing the relative level of demand uncertainty. Therefore, analysis of optimal inventory placement in different networks is relevant. It is also possible to analyze the case where multiple disruption levels are assumed. The current model has two scenarios: normal operations or disrupted operations. A more realistic case can be analyzed by expanding them to the multiple cases such as normal operations, modest disruption and / or significant disruption, each of which has different processing time and probability.

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