A DETERIORATING INVENTORY MODEL WITH LIMITED VEHICLE CAPACITY, STOCK DEPENDENT DEMAND AND UNAVAILABILITY SUPPLY

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ABSTRACT

Inventory model is one research topic that has been given attention intensively in the supply chain. There are two main costs for inventory, which are transport cost and inventory cost. Therefore, some buyers would like to apply a Vendor Managed Inventory (VMI) system where vendors handle the transportation and manage stocks at the buyer's hand. The problem is complex for some items, such as fruits and vegetables, in which the items deteriorate. The need for fruits and vegetables tends to be higher as the stock is high. Deteriorating inventory models have been developed in many years, however, only a few models considering vehicle capacity, carbon emission, deteriorating items, stock dependent demand, and unavailability supply. In this study, a deteriorating inventory model for multi items in one distribution with stock dependent demand is improved. On the other hand, fruits and vegetable stock are not consistently available, so lost sales costs should be examined. Environmental issues have been studied by many researchers. Therefore, we further consider the carbon emission yield in this model. Since the closed-loop solution can not be obtained, we employ a simple heuristic solution in Maple. A sensitivity analysis is employed to obtain some management insight. The sensitivity analysis indicates that the carbon emission tax rate can encourage decision-makers to increase order quantity and reduce carbon emission, but the policy should deal with many features that are recognized by decision-makers to make it useful.

Keywords: inventory, deteriorating, stock dependent demand, carbon emission

1. INTRODUCTION

Fresh food inventory problems are further complex than other item types, since they have some particular attributes. First, most fresh foods deteriorate and the amount diminishes in age. Second, the supply of the items is not stable. In the early harvest season, it is tough to get things that are mature enough and the price is costly. At the edge of the harvest season, the need goes to go down and many pieces are very mature. It is crucial to get a stable supply from the supplier at this stage. Third, the amount of items that looks attractive to place on the rack is declining since demand goes to decline and need to go to depend on the amount of stock. When the amount of stock on the rack is large, demand goes to be large. This study develops an inventory model for fresh food crops in the last harvest season.

One marketing plan to appeal to consumers to use money by adding more stock on a rack. Customer willingness for having a product depends on the amount of items on the rack. This is called stock dependent demand. Research on stock dependent demand inventory model was developed first by Gupta and Vrat (1986). Mandal and Phaudjar (1989) developed later an economic production quantity model for deteriorating items by considering stock-dependent consumption rate. An inventory deteriorating item model with stock dependent demand was developed by Baker and Urban (1988) and Pal et al. (1993) continued their work. Hou (2006) analyzed the effect of inflation and time discounting for deteriorating inventory problems with stock dependent demand. Lee and Dye (2012) developed a deteriorating inventory items model with stock dependent demand by controlling the deteriorating rate using various efforts in technology. They evaluated technology investment yield and save deteriorating rate reduction. Teng and Chang (2005) developed a production economic quantity model for deteriorating items. They concluded that great goods displayed in a grocery store could make more demands and pay off, however a lot of stock should be considered since the amount of stock on display is not good because too much stock gives a negative impression to customers. Later, Li et al. (2017) developed dynamic pricing and periodic order quantity model for deteriorating items and considering stock-dependent demand. Shortages and the backlogging are allowed, and the amount is volatile and depends on the period for the next replenishment point.

Although there is intensive research on deteriorating inventory models with stock dependent demand, only a few considering deteriorating inventory models with stock dependent demand and unavailability supply. The unavailability supply is relevant since the supply of some deteriorating items such as fruits and vegetables is not stable and supply is not consistently available. The lack of supply influenced by machine unavailability was considered by some research such as Sutapa and Widyadana (2014) and Al-Salamah (2018). The other significant issue is carbon emission since items delivery requires a truck that has a contribution to carbon emission. This paper is divided into four sections. In the first section, some relevant literature is introduced, and the contribution of the paper is shown. Some mathematical models are developed in Section 2, and a numerical example and sensitivity analysis are presented in Section 3 to give some management insight into the model. In the last section, some exciting conclusions are shown.

2. MODEL DEVELOPMENT

In this study, a deteriorating inventory model with stock dependent demand, unreliable supply, vehicle capacity constraint, and carbon emission cost is introduced. In practice, the model describes two conditions for products. When a firm orders Q units, the items will be used up by the consumer's demand and deteriorated rate until reach zero units at time $T1$. When inventory reaches zero, Q units are bought and fulfill the stock. However, there is a probability that the supplier cannot fulfill the requirement and delay delivery date for $T2$. Since there is a delivery lag, lost sales cost will arise.

Notations:

I_t = Inventory level at t period
β = stock sensitivity rate
d = demand rate

θ = deteriorating rate
K = setup cost
H = holding cost
S = lost sales cost
TC = total inventory cost
T_1 = replenishment time
T_2 = shortage time
T = total replenishment time
Cap_i = Capacity of vehicle i
TCT = total cost
$F^e{}_i$ = average emission from fuel combustion of vehicle i (kgCO_2/liter)
T_x = carbon emission tax ($/ton$CO_2$)
c_1 = average vehicle fuel consumption of vehicle i (liter/trip);
e_i = transportation emission cost of vehicle i ($/trip); $e_1 = c_1 F_e t_x$;

The inventory level for stock dependent demand deteriorating inventory can be denoted by the accompanying equation:

$$\frac{dI}{dt} + \theta I_t = -d(I(t)), \qquad 0 \le t \le T_1 \tag{1}$$

Where

$$d = \beta I(t) \tag{2}$$

One obtains:

$$I(t) = \frac{1}{\theta+\beta}(e^{(\theta+\beta)(T_1-t)} - 1) \qquad 0 \le t \le T_1 \tag{3}$$

Total inventory for $0 \le t \le T_1$ is

$$\int_0^{T_1} I(t)dt = \int_0^{T_1} \frac{1}{\theta+\beta}(e^{-(\theta+\beta)(T_1-t)} - 1)\,dt$$
$$= \frac{(e^{-(\theta+\beta)T_1} - \theta T_1 - T_1\beta - 1)}{(\theta+\beta)^2} \tag{4}$$

Since supply is not always available, therefore lost sales can take place when supply is not available when the stock becomes diminished to zero. So the total inventory cost for lost sales consists of setup cost, carbon emission cost, holding cost and lost of goodwill. The lost sales probability follows probability function ($f(t)$). The total inventory cost can be modeled as:

$$TC(T) = E\left[K + e_i + h\left(\int_0^{T_1} \frac{1}{\theta+\beta}(e^{-(\theta+\beta)(T_1-t)} - 1)\right) + S\,\frac{\int_{t=T_1}^{\infty} e^{-\beta t} f(t)dt}{\beta}\right] \tag{5}$$

Since $T_2 = \int_{t=T_1}^{\infty}(t-T_1)f(t)dt$, the total cost per unit time can be derived as follows:

$$TCT(T) = \frac{E\left[K + e_i + h\left(\int_0^{T_1} \frac{1}{\theta+\beta}(e^{-(\theta+\beta)(T_1-t)} - 1)\right) + S\,\frac{\int_{t=T_1}^{\infty} e^{-\beta t} f(t)dt}{\beta}\right]}{E\left[T_1 + \int_{t=T_1}^{\infty}(t-T_1)f(t)dt\right]} \quad (6)$$

with vehicle capacity constraint:

$$Q <= Cap_i \quad (7)$$

Where
$$Q = \frac{\beta}{\theta+\beta}(e^{(\theta+\beta)T_1} - 1) \quad (8)$$

In this study, we assume the supply availability time will follow a uniform distribution. Substitute uniform probability density function in (6), one has:

$$TCT(T) = \frac{K + e_i + h\left(\frac{(e^{-(\theta+\beta)T_1} - \theta T_1 - T_1\beta - 1)}{(\theta+\beta)^2}\right) + Sd\left(\frac{(b-T_1)^2}{2b}\right)}{T_1 + \frac{(b-T_1)^2}{2b}} \quad (9)$$

The optimal T_1 can be found by derivate (9) concerning T_1 and set the result to zero

$$\frac{dTCT(T_1)}{dT} = \frac{\left(K + e_i + h\left(\frac{(e^{-(\theta+\beta)T_1} - \theta T_1 - T_1\beta - 1)}{(\theta+\beta)^2}\right) + \frac{S\left(\beta^2 b^2 - 2T_1\beta^2 b + 2e^{-\beta b}(1+\beta b - T_1\beta) + \beta^2 T_1{}^2 - 2e^{-T_1\beta}\right)}{2\beta^3 b}\right)\left(1 - \frac{b-T_1}{b}\right)}{T_1 + \frac{(b-T_1)^2}{2b}}$$

$$- \frac{\left(h\left(\frac{(\theta+\beta-(\theta+\beta)e^{T_1(\theta+\beta)}}{(\theta+\beta)^2}\right) - \frac{S\left(-2\beta^2 b + 2T_1\beta^2 - 2e^{-\beta b}\beta + 2\beta e^{-T_1\beta}\right)}{2\beta^3 b}\right)}{T_1 + \frac{(b-T_1)^2}{2b}} = 0 \quad (10)$$

Since the closed-form solution of (10) cannot be derived, a simple heuristic from Maple is used to solve the model. Since there is vehicle constraint capacity, therefore steps below is applied:

Step 1: Find the optimal replenishment period using (11).

Step 2: Calculate quantity delivered by a truck in a single trip using (8) for all truck types and compare the quantity with truck capacity

Step 3: If the order quantity less or equal than truck capacity (7), the optimal replenishment time is derived. If not, the order quantity is equal to truck capacity and find the replenishment period

Step 4: Compare the total cost of all truck types and find the lowest cost.

3. A NUMERICAL EXAMPLE AND SENSITIVITY ANALYSIS

A numerical example is shown to illustrate the model. We use K = 100, a = 0.05, h = 1, S = 10 and b = 1 and stock dependent parameter β = 10. Some emission data are derived from Daryanto (2019) and the truck emission data ad carbon tax emission is shown in Table 1. Since the closed form solution cannot be found, the model is solved using a numerical method in Maple.

The result for each transportation type is presented in Table 2. The optimal ordering quantity for truck type 1 is 1093.3, truck type 2 = 1109.3 and truck type 3 is 1141.3, however the optimal ordering quantity for truck type 1 and truck type 2 is greater than the truck capacity. Therefore the ordering quantity for truck 1 is equal to 500 and the ordering quantity for truck 2 is 1000. The optimal total cost is derived when using truck type 2 with ordering time (T^*) is equal to 0.619 and the total cost is 158.5. The detail of the optimal solution is shown in Table 2.

Table 1. Truck emission data

Truck type	**Truck capacity**	**Average emission** **ton**CO_2/liter	**Carbon emission tax** **$/ ton**$CO_2$	**Average fuel consumption** **liter/trip**
1	500	2.6 x 10^{-3}	75	27.5
2	1000	2.6 x 10^{-3}	75	35
3	1500	2.6 x 10^{-3}	75	50

Table 2. Decisions for each truck type

Truck type	**Truck capacity**	***Q****	***T****	**Total cost**
1	500	1093.3	0.619	159.551
2	1000	1109.3	0.688	158.499
3	1500	1141.3	0.701	162.361

A sensitivity analysis is conducted using varies of carbon emission tax and let other parameters in the same values. The sensitivity analysis result is shown in Table 3-6.

Table 3. Solution for carbon emission tax = 60

Truck type	**Truck capacity**	***Q****	***T****	**Total cost**
1	500	1081.6	0.619	158
2	1000	1094.35	0.688	**156.646**
3	1500	1119.93	0.699	159.744

Table 4. Solution for carbon emission tax = 67.5

Truck type	**Truck capacity**	***Q****	***T****	**Total cost**
1	500	1087.43	0.619	158.78
2	1000	1101.81	0.688	**157.573**
3	1500	1130.59	0.700	161.053

Table 5. Solution for carbon emission tax = 82.5

Truck type	Truck capacity	*Q**	*T**	Total cost
1	500	1099.15	0.619	160.331
2	1000	1116.73	0.688	**159.426**
3	1500	1151.92	0.70199	163.668

Table 6. Solution for carbon emission tax = 90

Truck type	Truck capacity	*Q**	*T**	Total cost
1	500	1105.55	0.619	161.0981
2	1000	1124.19	0.688	**160.367**
3	1500	1162.59	0.703	164.974

Table 3-6 shows that in varies of carbon emission tax, the result to use truck type 2 is not changed, and therefore in this case carbon emission tax is not sensitive to the truck type decision. However the total cost increase as the carbon emission tax increase. Since the optimal ordering quantity for truck type 2 bigger than the truck capacity, therefore changing the carbon emission tax will not change the ordering quantity. However, if the optimal ordering quantity is fewer than truck capacity, changing of carbon emission tax can modify the ordering quantity. Result of truck type 3 shows that the ordering quantity increase as a carbon tax emission increase. Higher carbon tax emission can encourage decision-makers to increase ordering quantity, therefore the effect of carbon emission can be decreased. However, the carbon emission tax is not consistently rewarding to encourage business decision-makers to adjust their decision to cut carbon emission by increasing ordering quantity. There are many criteria should be dealt with by the decision-maker that makes companies does not support carbon emission reduction, and in this instance, is the truck capacity. It is essential to set carbon emission tax correctly, so the decision is not only caused some business spend more money, but the purpose of cutting carbon emission can be carried out.

4. CONCLUSION

In this study, a deteriorating inventory model with stock dependent demand, unreliable supply, truck capacity constraint, and carbon emission tax is developed. The model is solved using a heuristic method based on Maple since a closed-loop solution can nod derived. A sensitivity analysis is conducted to verify model and illustrate the effect of carbon emission tax on the ordering quantity.

The sensitivity analysis indicates that the ordering quantity is not changing as the carbon tax emission tax increase since the optimal ordering quantity bigger than truck capacity. However, if there is no truck capacity constraint, the ordering quantity increase as the carbon emission tax increase. By increasing the ordering quantity, the effect of carbon emission can be reduced.

Therefore carbon emission tax policy can reduce carbon emission, but many parameters should be considered making it effective. The model assumes that truck only delivers a single item, so truck type 3 delivers items less than capacity and truck-only send to one buyer. The model can be improved by considering the multi-item and routing problem.

5. REFERENCES

Al-Salamah M. (2018). Economic production quantity with the presence of imperfect quality and random machine breakdown and repair based on the artificial bee colony heuristic. *Applied mathematical Modelling*, 63, 68-83.

Baker R.C., and Urban T.L. (1988). A deterministic inventory system with an inventory-level-dependent demand rate. *J. Opl. Res. Soc.*, 39(9), 823-831.

Daryanto Y. Wee H.M., Widyadana G.A. (2019). Low carbon supply chain coordination for imperfect quality deteriorating items. *Mathematics*, 3(7), 234.

Gupta, R., Vrat, P. (1986). Inventory model for stock-dependent consumption rate. *Opsearch*, 23, 19-24.

Hou K. L. (2006). An inventory model for deteriorating items with stock-dependent consumption rate and shortages under inflation and time discounting. *European Journal of Operational Research,* 168, 463-474.

Lee Y.P. and Dye C.Y. (2012). An inventory model for deteriorating items under stock-dependent demand and controllable deterioration rate. *Computers & Industrial Engineering*, 63, 474-482.

Li Y., Zhang S., and Han J. (2017). Dynamic pricing and periodic ordering for a stochastic inventory system with deteriorating items. *Automatica*, 76, 200-213.

Mandal B.N. and Phaudjar S. (1989). An inventory model for deteriorating items and stock-dependent consumption rate. *J. OPl. Res. Soc*., 40, 483-488.

Pal S., Goswami A., and Chaudhuri K.S. (1993). A deterministic inventory model for deteriorating items with stock-dependent demand rate. *International Journal of Production Economics*, 32, 291-299.

Sutapa N., Widyadana G.A. (2014). The effect of unreliable machine for two echelons deteriorating inventory model, *Jurnal Teknik Industri*, 16(2), 107-112.

Teng J.T., and Chang C.T. (2005). Economic production quantity models for deteriorating items with price- and stock-dependent demand. *Computers & Operations Research*, 32, 297-308.