SUPPLY CHAIN COORDINATION UNDER VENDOR MANAGED INVENTORY SYSTEM CONSIDERING CARBON EMISSION FOR IMPERFECT QUALITY DETERIORATING ITEMS

Agustina Viani Trinanda Adi Nugroho
Department of Industrial and Systems Engineering, Chung Yuan Christian University, Taoyuan City, Taiwan 32023, R.O.C., E-mail: agustinavianitrinanda@gmail.com

Hui Ming, Wee
Department of Industrial and Systems Engineering, Chung Yuan Christian University, Taoyuan City, Taiwan 32023, R.O.C., E-mail: weehm@cycu.edu.tw

ABSTRACT

Inventory management and transportation are the main activities in supply chains that generate carbon emissions in many industries. The industries will produce carbon emission significantly to keep maintaining their item well, especially deteriorating items. Recently, many researchers focus on reducing carbon emissions regarding carbon policy that set in the industries. This study considers supply chain coordination between single supplier and buyer, mixing with Vendor Managed Inventory system to manage inventory decision considering carbon emission and imperfect quality deteriorating items. The transportation scenario of supplier delivering the product to the buyer using two different cases, the first is when the supplier transports the products by single-setup-single-delivery policy, and the second is single-setup-multiple-delivery policy. Carbon emission cost is also calculated under carbon tax policy. The aim of this study is to minimize the total cost and reduce the total carbon emission considering the optimum delivery quantity and the number of deliveries per production cycle. Numerical example and the sensitivity analysis will be provided to show the solution of the proposed model.

Keywords: Supply Chain, Vendor Managed Inventory, Carbon Emission, Imperfect Quality, Deteriorating Items.

1. INTRODUCTION

Nowadays, there are many carbon emission restriction policies which made enterprises and researchers focus on reducing carbon emissions. They develop sustainable supply chain management systems especially inventory models considering carbon emissions. Inventory management and transportation are the main activities in supply chains that generate carbon emissions in many industries. Road transportation causing costs to increase for firms that deliver and gather the products, and it can be a source of bad environmental impacts such as CO_2 emissions (Palmer, 2007).

This study proposes supply chain inventory models that consider carbon emissions cost and defective items under Vendor Managed Inventory (VMI) systems. VMI promotes a win-win scenario for both vendors and customers. Vendors can maintain distribution cost as they have the freedom to combine and coordinate shipments for different customers. Meanwhile, customers save on resources since they do not need to exert effort in the control and management of inventories (Coelho et al., 2013). In this paper, we focused on two-echelon supply chain between a supplier and buyer.

Many researches that consider carbon emissions while trying to implement VMI systems have been undergone. Lee and Ren (2011) provides a simple stochastic model that captures the benefits of VMI in a global environment. Setak and Daneshfar (2014) consider the supply chain collaboration via a VMI systems and provide EOQ model for a two level supply chain. This study also considers stock-dependent demand and constant deterioration and backlogging rates, to examine the inventory management for VMI and non-VMI supply chains. Jiang et al. (2015) proposes a green vendor managed inventory model with a supplier and a manufacturer under a carbon emissions trading mechanism. This paper finds that in the green VMI model the decision for the supplier whether to sell or to buy carbon credit depends on the carbon cap. Mateen and Chatterjee (2015) present an approach which shows supply chain coordination through VMI that can lead to significant economic benefits and at the same time lead to reduction in greenhouse gas emissions. On the other hand, Bai, et al. (2019) investigate the effects of carbon emission reduction on a supply chain with one manufacturer and two competing retailers for deteriorating products under VMI.

This study focuses on inventory modeling under VMI system that contains imperfect quality deteriorating items. Dealing with imperfect quality or defective items affects the amount of carbon emissions because processing these will add to the total carbon emissions accounting for the holding costs; moreover, the loss due to imperfect quality and deterioration also drives the supplier to produce more products to satisfy customer demand per period; therefore carbon emissions will increase in the production process by holding the inventory and by the distribution of the product. This study attempts to extend the mathematical model from Daryanto, Wee, and Widyadana (2019) to mix with VMI systems. In this paper, we focused on two-echelon supply chain between a supplier and buyer and considering two cases of transportation policy. First case is when supplier transports the product to the buyer under single-setup-single-single-delivery (SSSD) policy. According to Sarkar et al. (2016), under SSSD policy, all products are produced in a single setup and then supplier transported the products to the buyer in a single delivery. The second is when the products are produced in single-setup-multiple-delivery (SSMD) policy, which is the products are produced in a single setup but it delivers to the buyer in multiple deliveries.

This research considers carbon emissions and suggests ordering and inventory decisions for deteriorating imperfect quality items. The paper is organized as follows: Section 1 is the introduction, followed by literature review in the section 2; section 3 presents the problem assumptions and list of notations; section 4 establishes the model development; section 5 shows the numerical example; and the last sections are conclusion and suggestion for future research.

2. LITERATURE REVIEW

Supply chain coordination investigated in a few works of literature. Goyal (1997) studied the integrated inventory policy to achieve economic saving when the supplier and the buyer decided to cooperate. Wang et al. (2011) considered four separate types of supply chain inventory policies: individual isolated policy, buyer's perspective policy, manufacturer's perspective policy, and integrated policy. Glock (2012) made a literature review of the integrated supply chain.

Vendor Managed Inventory is an approach for inventory management and order accomplishment. VMI involves collaboration between suppliers and customers in using different ordering processes to change the traditional ones. VMI has a goal to provide mutual business objectives for both suppliers and the customers. Direct results such as the increase in the number of improved inventory turn-outs, improved services, and increased sales determine the business value. VMI mostly focuses on finding a flexible strategy for replenishment, shipment coordination, and the impact of information sharing performance (Wong et al., 2009).

The supplier manages the buyer's inventory and decides the period and quantity needed to optimally replenish the customer goods. VMI partnership enables the suppliers to make decisions regarding inventory replenishment for buyers (Rana et al., 2015). The supplier receives electronic data via internet that tells the supplier about the customer's sales and stock levels. The supplier can view every item that the customer carries as well as true point of sale data in most cases. The supplier's liability is to create and maintain the inventory plan. Under VMI, the supplier generates the order for replenishment VMI inventory.

In production process there are certain outputs that contain defective items, because the process is not always perfect. Since this study using vendor managed inventory system, inspection process will be complied with the supplier or vendor. Imperfect quality is studied by Bazan et al. (2014) that explain about the vendor decisions that conducts inspection towards imperfect quality items, such as scrap off, salvage at a discounted price, and rework. Other researchers bring the effect of imperfect quality items into integrated vendor and buyer. Huang (2002) considered the imperfect quality and assumed that the supplier provides a product warranty for the defective items. The inspection is 100% conducted by supplier. Hsu (2012) developed a single vendor single retailer production inventory model in which defective items are returned to the vendor immediately under unequal shipment size. Sarkar et al. (2017) studied the integrated inventory model with transportation cost and two-stage inspection by the vendor.

3. PROBLEM ASSUMPTIONS AND NOTATIONS

This research considers a supplier-buyer supply chain that produces one type of item. This research is an extension of Daryanto et al.'s (2019) mathematical model by mixing with VMI systems. Using these systems, the supplier takes roles for creating and maintaining the inventory plans of both suppliers' and buyers. According to the information shared on the actual inventory level, supplier will determine the time and the amount of items that buyer needed. Both the supplier and the buyer consider the carbon emission cost in their decision to fulfill the carbon tax regulation. This study develops two models considering two different cases of transportation policy. First, the supplier produces and delivers the products in a single setup and delivery. Second is when the supplier produces the products in single setup but deliver the products to the buyer in multiple deliveries. Thus, they produce nQ units of item per production cycle to reduce setup time and cost. Then, they deliver the item in equal lot sizes and constant time intervals (Cao et al., 2007). This study consider following assumptions:

1. The supplier's production rate and the buyer's demand rate are known and constant.
2. The replenishment is instantaneous.
3. The deterioration rate for both items in supplier's and buyer's inventory are equal and constant.
4. The defective percentage (u) has a uniform distribution where $0 \leq \alpha < \beta < 1$.
5. Good products are always available during the quality inspection as $x > D$.
6. The supplier performs 100% quality inspection to ensure an excellent service.
7. The inspection cost per cycle is constant.
8. Carbon emissions are the outcomes of fuel and electricity consumption during transportation and holding the inventory holding.

Shortage is not considered.

The notations used in developing the mathematical models are provided in Table 1.

4. MODEL DEVELOPMENT

This section provides model development for two transportation policy cases.

Table 1. List of notations

Symbol	Definition
D	demand rate (unit/year);
P	production rate (unit/year);
R	production quantity; $R = PT_1$;
θ	deterioration rate; $(0 \leq \theta < 1)$;
u	the probability of defective products per delivery lot size;
x	quality screening rate (unit/year);
i_c	fixed quality inspection cost ($/cycle);
u_c	unit inspection cost ($/unit);
c	buyer's ordering cost ($/order);
h_d	buyer's holding cost ($/unit/year);
d_d	buyer's deteriorating cost ($/unit);
s	supplier's setup cost ($/order);
h_p	supplier's holding cost ($/unit/year);
d_p	supplier's deteriorating cost ($/unit);
p_m	Supplier's production cost ($/unit);
P_e	Supplier's carbon emissions from production activities (tonCO_2/unit)
t_f	supplier's fixed transportation cost per delivery ($/delivery);
t_v	fuel price for supplier's variable transportation cost ($/liter);
d	distance traveled from vendor to buyer (km);
w	product weight (ton/unit);
c_1	average vehicle fuel consumption when empty (liter/km);
c_2	average additional fuel consumption per ton of load (liter/km/ton);
T_x	carbon emission tax ($/ton$CO_2$);
F_e	average emissions from fuel combustion (tonCO_2/liter);
E_e	average emissions from electricity generation (tonCO_2/kWh);
e_1	transportation emission cost ($/km); $e_1 = c_1F_eT_x$;
e_2	average additional transportation emission cost per unit product ($/unit/km); $e_2 = c_2wF_eT_x$;
e_c	average warehouse energy consumption per unit product (kWh/unit/year);
w_e	warehouse emissions cost per unit product ($/unit/year); $w_e = e_cE_eT_x$;
T	cycle length;
T_1	production period for the supplier in each cycle;
T_2	nonproduction period for the supplier in each cycle;
$I_p(t)$	supplier's inventory level at time t;
$I_{pd}(t)$	supplier's inventory for defective products at time t;
$I_d(t)$	buyer's inventory level at time t;
ETC_d	buyer's expected total cost per year ($/year);
ETC_p	supplier's expected total cost per year ($/year);
ETC	joint expected total cost per year ($/year);
Decision Variables	
Q	delivery lot size (unit);
n	number of deliveries per order (positive integer).

4.1. Model Development with Single-Setup-Single-Delivery (SSSD) Policy

In this sub-section presents model development that considering a single setup and a single delivery of the products. The supplier will produce a type of deteriorating items and deliver the finished products produced during each production cycle to the buyer. This research also considers about imperfect quality items. According to Daryanto et al. (2019), after inspection done by the supplier, the defective products will be sold at a discounted price to the secondary market. Figure 1 illustrates the inventory level for both the supplier (I_p) and buyer (I_d).

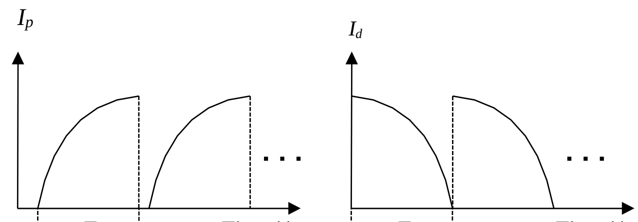

Figure 1. Inventory level of the supplier under SSSD policy.

Buyer Cost and Emission Function

Referring to Yang and Wee (2003), the inventory function is as follows:

$$I_d(t) = \frac{D}{\theta}\left(e^{\theta(T-t)} - 1\right), \quad 0 \;\; t \leq T \tag{1}$$

and

$$Q = I_d(0) = \frac{D}{\theta}\left(e^{\theta T} - 1\right) \tag{2}$$

During each replenishment cycle, the sum of inventory holding cost of the buyer will be:

$$\frac{h_d}{T}\int_0^T I_d(t)\ dt = \frac{h_d\left(D\left(\frac{e^{\theta T}-1}{\theta} - T\right)\right)}{T\theta} \tag{3}$$

The buyer's total cost is the sum of the holding, deteriorating, and emission costs. The buyer's expected total cost will be:

$$\frac{h_d\left(D\left(\frac{e^{\theta T}-1}{\theta} - T\right)\right)}{T\theta} + \frac{d_d\left(D\left(\frac{e^{\theta T}-1}{\theta} - DT\right)\right)}{T} + \frac{w_e\left(D\left(\frac{e^{\theta T}-1}{\theta} - T\right)\right)}{T\theta} \tag{4}$$

Supplier Cost and Emission Function

Due to some percentage of defective products, the production rate of the perfect product is $(1-u)P$. The supplier's setup cost per year is □. The supplier's inspection cost consider a fixed Inspection cost per cycle (i_c) and unit inspection cost (u_c). The total number of products being Produced per production cycle is PT_1, therefore the supplier's inspection cost per year is:

$$\frac{i_c}{T} + \frac{u_c P T_1}{T} \tag{5}$$

The supplier's transportation cost is:

$$\frac{\left(t_f + 2dc_1t_v + d\frac{D}{\theta}\left(e^{\theta T} - 1\right)wc_2t_v\right)}{T} \tag{6}$$

t^f is the fixed transportation cost per delivery. The second part calculates the transportation cost of an empty truck. The distance multiplied by two since the truck goes from the supplier to the buyer and the goes back. The third part is the transportation cost for the truckload, which depends on the delivery distance and quantity, product weight, additional fuel consumption per ton per km, and the fuel price. Therefore, the amount of the supplier's carbon emission per year as the result of transportation activity can be derived as follows:

$$T\left(2dc_1 + d\frac{D}{\theta}(e^{\theta T} - 1)wc_2\right)F_e \quad (7)$$

From Figure 2, the inventory differential equations during the production period are:

$$dI_{p1}(t_1) = ((1-u)P - D)dt_1 - \theta I_{p1}(t_1)dt_1, \quad 0 \leq t_1 \leq T_1 \quad (8)$$

$$I_{p1}(t_1) = \frac{(1-u)P - D}{\theta}(1 - e^{-\theta t_1}), \quad 0 \leq t_1 \leq T_1 \quad (9)$$

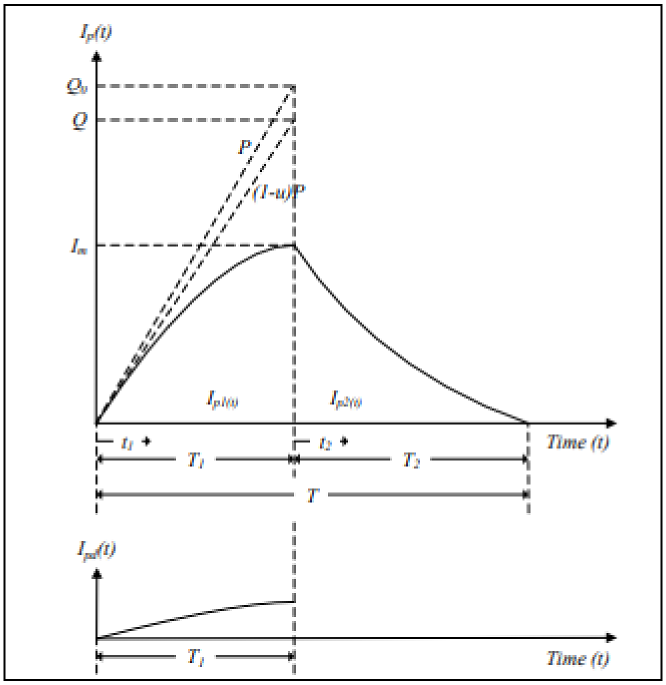

Figure 2. Production and non-production period of supplier's inventory

And the supplier's inventory product during the non-production period:

$$dI_{p2}(t_2) = -Ddt_2 - \theta I_{p2}(t_2)dt_2, \quad 0 \leq t_2 \leq T_2 \quad (10)$$

$$I_{p2}(t_2) = \frac{D}{\theta}(e^{\theta(T_2 - t_2)} - 1), \quad 0 \; t_2 \leq T_2 \quad (11)$$

Following Misra's (1975) approximation:

$$T_1 \approx \frac{D}{(1-u)P - D}T_2\left(1 + \frac{1}{2}\theta T_2\right) \quad (12)$$

$$T \approx \frac{T_2}{(1-u)P - D}\left((1-u)P + \frac{1}{2}D\theta T_2\right) \quad (13)$$

Therefore, the supplier's inventory for good products becomes:

$$\int_0^{T_1}\frac{(1-u)P - D}{\theta}(1 - e^{-\theta t_1})dt_1 + \int_0^{T_2}\frac{D}{\theta}(e^{\theta(T_2 - t_2)} - 1)dt_2 - \left[\frac{D}{\theta}\left(\frac{1}{\theta}(e^{\theta T} - 1) - T\right)\right] \quad (14)$$

Besides, there is an inventory of defective products. From Figure 2, the inventory differential equation for the defective products is:

$$dI_{pd}(t_1) = uPdt_1 - \theta I_{pd}(t_1)dt_1, \quad 0 \le t_1 \le T_1 \tag{15}$$

$$I_{pd}(t_1) = \frac{uP}{\theta}(1 - e^{-\theta t_1}), \quad 0 \le t_1 \le T_1 \tag{16}$$

Therefore, the supplier's inventory of the defective products becomes:

$$\int_0^{T_1} \frac{uP}{\theta}(1 - e^{-\theta t_1})dt_1 \tag{17}$$

Hence, the supplier's holding cost per year is:

$$\frac{h_p}{T}\left(\frac{(1-u)P - D}{\theta}T_1 + \frac{(1-u)P - D}{\theta^2}(e^{-\theta T_1} - 1) - \frac{DT_2}{\theta} - \frac{D}{\theta^2}(1 - e^{\theta T_2}) - \left(\frac{D}{\theta}\left(\frac{1}{\theta}(e^{\theta T} - 1) - T\right)\right) + \frac{uPT_1}{\theta} + \frac{uP}{\theta^2}(e^{-\theta T_1} - 1)\right) \tag{18}$$

Based on Equations (6) and (14) the supplier's carbon emission cost and the total expected carbon emissions per year can be calculated as follows:

$$T\left(2de_1 + d\frac{D}{\theta}\left(e^{\frac{\theta T}{n}} - 1\right)e_2\right) + \frac{w_e}{T}\left(\frac{(1-u)P-D}{\theta}T_1 + \frac{(1-u)P-D}{\theta^2}(e^{-\theta T_1} - 1) - \frac{DT_2}{\theta} - \frac{D}{\theta^2}(1 - e^{\theta T_2}) - \left(\frac{D}{\theta}\left(\frac{1}{\theta}(e^{\theta T} - 1) - T\right)\right) + \frac{uPT_1}{\theta} + \frac{uP}{\theta^2}(e^{-\theta T_1} - 1)\right) \tag{19}$$

$$\begin{aligned} ETE_p = {} & T\left(2dc_1 + d\frac{D}{\theta}(e^{\theta T} - 1)wc_2\right)F_e \\ & + \frac{e_cE_e}{T}\left(\frac{(1-u)P - D}{\theta}T_1 + \frac{(1-u)P - D}{\theta^2}(e^{-\theta T_1} - 1) - \frac{DT_2}{\theta} \right. \\ & \left. - \frac{D}{\theta^2}(1 - e^{\theta T_2}) - \left(\frac{D}{\theta}\left(\frac{1}{\theta}(e^{\theta T} - 1) - T\right)\right) + \frac{uPT_1}{\theta} + \frac{uP}{\theta^2}(e^{-\theta T_1} - 1)\right) \\ & + e_cE_e\left(T\frac{D}{\theta}\left(\frac{1}{\theta}(e^{\theta T} - 1) - T\right)\right) \end{aligned} \tag{20}$$

On the other hand, production cost is calculated by:

$$\frac{(Pm + Pe\,Tx)T_1P}{T} \tag{21}$$

The number of deteriorated items in the supplier's inventory is the total production during the period T_1, minus the total products delivered to the buyer and the inventory of the defective products; therefore, the supplier's deteriorating cost per year is:

$$\frac{d_p}{T}\left((1-u)PT_1 - \left(\frac{D}{\theta}(e^{\theta T} - 1)\right) + \left(uPT_1 - \frac{uP}{\theta}(1 - e^{-\theta T_1})\right)\right) \tag{22}$$

The supplier's expected total cost is sum of set up, inspection, transportation, holding inventory, production, carbon emission, and deteriorating cost, which will be:

$$
\begin{aligned}
ETC_p = {} & \frac{s}{T} + \frac{i_c}{T} + \frac{u_c P T_1}{T} + \frac{\left(t_f + 2dc_1 t_v + d\frac{D}{\theta}\left(e^{\theta T} - 1\right) w c_2 t_v\right)}{T} \\
& + T\left(2de_1 + d\frac{D}{\theta}\left(e^{\theta T} - 1\right)e_2\right) \\
& + \frac{(h_p + w_e}{T}\left(\frac{(1 - E[u])P - D}{\theta}T_1 + \frac{(1 - E[u])P - D}{\theta^2}\left(e^{-\theta T_1} - 1\right) - \frac{DT_2}{\theta}\right. \\
& - \frac{D}{\theta^2}\left(1 - e^{\theta T_2}\right) - \left(\frac{D}{\theta}\left(\frac{1}{\theta}\left(e^{\theta T} - 1\right) - T\right)\right) + \frac{E[u]PT_1}{\theta} \\
& \left. + \frac{E[u]P}{\theta^2}\left(e^{-\theta T_1} - 1\right)\right) \\
& + \frac{d_p}{T}\left((1 - E[u])PT_1 - \left(\frac{D}{\theta}\left(e^{\theta T} - 1\right)\right)\right. \\
& \left. + \left(E[u]PT_1 - \frac{E[u]P}{\theta}\left(1 - e^{-\theta T_1}\right)\right)\right) + \frac{(Pm + Pe\,Tx)T_1 P}{T}
\end{aligned}
\qquad (23)
$$

4.2. Model Development with Single-Setup-Multi-Delivery (SSMD) Policy

This sub-section provides model development under a single setup and multiple deliveries. The buyer orders *n* deliveries of equal lot size (Q) per cycle. They produce *n*Q units of item per production cycle to reduce setup time and cost. According to Sarkar et al. (2016), when the supplier adopts SSMD policy, the number of the transportation increases. As a benefit, it can save holding cost of the buyer. Therefore, there is a tradeoff between the transportation cost and holding cost. It can affect carbon emission that is produced by the transportation and the holding inventory activities. Figure 3 shows the level of inventory of the supplier (I_p) and the buyer (I_d), include the supplier's inventory of defective products (I_{pd}). The figure also shows I_d decreased with demand and deterioration during the *T/n* period.

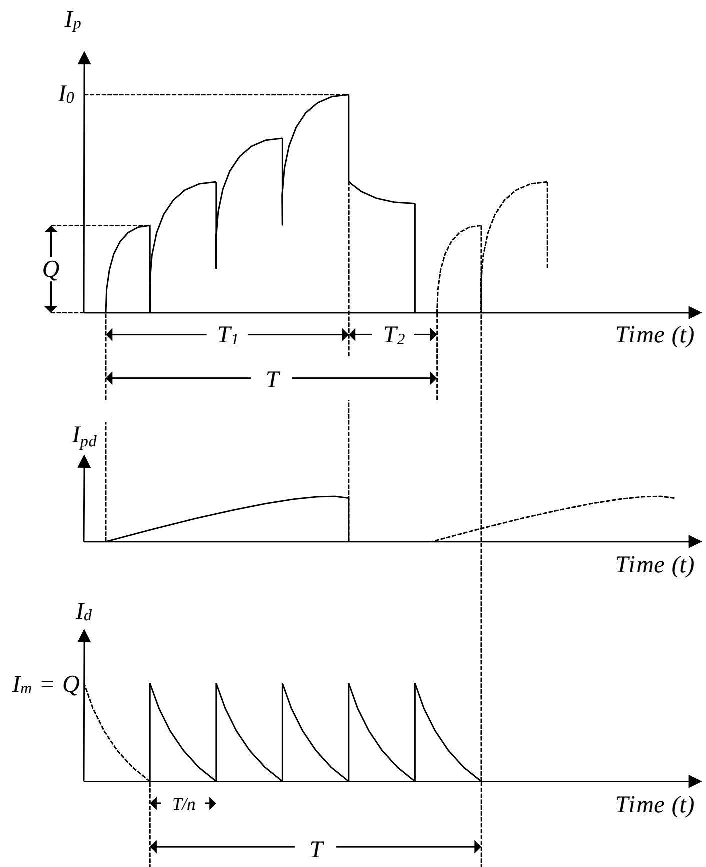

Figure 3. Inventory level of the supplier and buyer under SSMD policy.

Buyer Cost and Emission Function

During each replenishment cycle, the sum of inventory holding cost and deterioration cost of the buyer will be:

$$\left(h_d \frac{n}{T} + \theta d_d \frac{n}{T}\right) \int_0^{T/n} I_d(t)\ dt \tag{24}$$

Emission of the buyer is produced by holding the inventory, therefore the emission cost is:

$$w_e \int_0^{T/n} I_d(t)\ dt \tag{25}$$

The buyer's total cost is sum of the inventory, deterioration, and emission cost. The expected total cost per year (ETC_d) becomes:

$$ETC_d = (h_d + w_e)\frac{n}{T}\left(\frac{D}{\theta}\left(\frac{1}{\theta}\left(e^{\frac{\theta T}{n}} - 1\right) - \frac{T}{n}\right)\right) + d_d \frac{n}{T}\left(\frac{D}{\theta}\left(e^{\frac{\theta T}{n}} - 1\right) - \frac{DT}{n}\right) \tag{26}$$

Supplier Cost and Emission Function

For the SSMD policy, the supplier has same setup and inspection cost with the SSSD one, which in Equations (5) and (6). But for the carbon emission produced by transportation activity, the carbon emission function will be:

$$\frac{n}{T}\left(t_f + 2dc_1t_v + d\frac{D}{\theta}\left(e^{\frac{\theta T}{n}} - 1\right)wc_2t_v\right) \tag{27}$$

Then, the supplier's inventory for good products follows Figure 2 for the production and non-production period, therefore the equations becomes:

$$\int_0^{T_1}\frac{(1-u)P-D}{\theta}\left(1-e^{-\theta\, t_1}\right)dt_1 + \int_0^{T_2}\frac{D}{\theta}\left(e^{\theta\,(T_2-t_2)}-1\right)dt_2 - n\left[\frac{D}{\theta}\left(\frac{1}{\theta}\left(e^{\frac{\theta T}{n}}-1\right)-\frac{T}{n}\right)\right] \tag{28}$$

The supplier's holding cost per year is:

$$\frac{h_p}{T}\left(\frac{(1-u)P-D}{\theta}T_1 + \frac{(1-u)P-D}{\theta^2}\left(e^{-\theta T_1}-1\right) - \frac{DT_2}{\theta} - \frac{D}{\theta^2}\left(1-e^{\theta T_2}\right) - n\left(\frac{D}{\theta}\left(\frac{1}{\theta}\left(e^{\frac{\theta T}{n}}-1\right)-\frac{T}{n}\right)\right) + \frac{uPT_1}{\theta} + \frac{uP}{\theta^2}\left(e^{-\theta T_1}-1\right)\right) \tag{95}$$

The supplier's carbon emission cost per year can be calculated as follows:

$$\frac{n}{T}\left(2de_1 + d\frac{D}{\theta}\left(e^{\frac{\theta T}{n}}-1\right)e_2\right) + \frac{w_e}{T}\left(\frac{(1-u)P-D}{\theta}T_1 + \frac{(1-u)P-D}{\theta^2}\left(e^{-\theta T_1}-1\right) - \frac{DT_2}{\theta} - \frac{D}{\theta^2}\left(1-e^{\theta T_2}\right) - n\left(\frac{D}{\theta}\left(\frac{1}{\theta}\left(e^{\frac{\theta T}{n}}-1\right)-\frac{T}{n}\right)\right) + \frac{uPT_1}{\theta} + \frac{uP}{\theta^2}\left(e^{-\theta T_1}-1\right)\right) + (h_d + w_e)\frac{n}{T}\left(\frac{D}{\theta}\left(\frac{1}{\theta}\left(e^{\frac{\theta T}{n}}-1\right)-\frac{T}{n}\right)\right) \tag{30}$$

and the total expected carbon emissions per year denoted by ETE:

$$ETE_p = \frac{n}{T}\left(2dc_1 + d\frac{D}{\theta}\left(e^{\frac{\theta T}{n}}-1\right)wc_2\right)F_e + \frac{e_cE_e}{T}\left(\frac{(1-u)P-D}{\theta}T_1 + \frac{(1-u)P-D}{\theta^2}\left(e^{-\theta T_1}-1\right) - \frac{DT_2}{\theta} - \frac{D}{\theta^2}\left(1-e^{\theta T_2}\right) - n\left(\frac{D}{\theta}\left(\frac{1}{\theta}\left(e^{\frac{\theta T}{n}}-1\right)-\frac{T}{n}\right)\right) + \frac{uPT_1}{\theta} + \frac{uP}{\theta^2}\left(e^{-\theta T_1}-1\right)\right) + e_cE_e\left(\frac{n}{T}\frac{D}{\theta}\left(\frac{1}{\theta}\left(e^{\frac{\theta T}{n}}-1\right)-\frac{T}{n}\right)\right) \tag{31}$$

The number of deteriorated items in the supplier's inventory is the total production during the period T_1, minus the total products delivered to the buyer and the inventory of the defective products; therefore, the supplier's deteriorating cost per year is:

$$\frac{d_p}{T}\left((1-u)PT_1 - n\left(\frac{D}{\theta}\left(e^{\frac{\theta T}{n}}-1\right)\right) + \left(uPT_1 - \frac{uP}{\theta}\left(1-e^{-\theta\,T_1}\right)\right)\right) \tag{32}$$

Considering additional the production cost based on Equations (21), the supplier's expected total cost per year is:

$$
\begin{aligned}
ETC_p = {} & \frac{s}{T} + \frac{i_c}{T} + \frac{u_c P T_1}{T} + \frac{n}{T}\left(t_f + 2dc_1 t_v + d\frac{D}{\theta}\left(e^{\frac{\theta T}{n}} - 1\right) w c_2 t_v\right) \\
& + \frac{n}{T}\left(2de_1 + d\frac{D}{\theta}\left(e^{\frac{\theta T}{n}} - 1\right) e_2\right) \\
& + \frac{(h_p + w_e)}{T}\left(\frac{(1 - E[u])P - D}{\theta} T_1 + \frac{(1 - E[u])P - D}{\theta^2}\left(e^{-\theta T_1} - 1\right) - \frac{DT_2}{\theta}\right. \\
& \left. - \frac{D}{\theta^2}\left(1 - e^{\theta T_2}\right) - n\left(\frac{D}{\theta}\left(\frac{1}{\theta}\left(e^{\frac{\theta T}{n}} - 1\right) - \frac{T}{n}\right)\right) + \frac{E[u]PT_1}{\theta} + \frac{E[u]P}{\theta^2}\left(e^{-\theta T_1} - 1\right)\right) \qquad (33) \\
& + (h_d + w_e)\frac{n}{T}\left(\frac{D}{\theta}\left(\frac{1}{\theta}\left(e^{\frac{\theta T}{n}} - 1\right) - \frac{T}{n}\right)\right) + \frac{(Pm + Pe\,Tx)T_1 P}{T} \\
& + \frac{d_p}{T}\left((1 - E[u])PT_1 - n\left(\frac{D}{\theta}\left(e^{\frac{\theta T}{n}} - 1\right)\right) + \left(E[u]PT_1 - \frac{E[u]P}{\theta}\left(1 - e^{-\theta T_1}\right)\right)\right)
\end{aligned}
$$

Integrated Total Cost Function

Before do the methodology and solution search, sum the Equations (5) and Equations (24) to find the ETC. Using Taylor's series expansion for a small value of $\theta T/n$, θT_1, and θT_2, we can solve the cost function by assuming e^x as $1 + x + x^2/2 + x^3/6$. This methodology is done to determine the optimal number of deliveries (n*) that minimize the expected total cost function *ETC*.

5. NUMERICAL EXAMPLE

The values of the parameters are considered by adopting data from Yang and Wee (2000), Hariga et al. (2017), Tiwari et al. (2018) as P = 2,000,000 units/year, D = 500,000 units/year, x = 1,725,000 unit/year, i_c = \$500/delivery, u_c = \$0.5/unit, c = \$2,000/order, s = \$100,000/setup, h_d = \$40/unit/year, h_p = \$40/unit/year, d_d = \$600/unit, d_p = \$400/unit, θ = 0.1, d = 100 km, t_f = \$1000/delivery, t_v = \$0.75/liter, w = 0.01 ton/unit, c_1 = 27 L/100 km, c_2 = 0.57 L/100 km/ton truckload, e_c = 1.44 kWh/unit/year, T_x = \$75/tonCO_2, Pm=\$10/unit, Pe= 10 kgCO_2/unit, $F_e = 2.6 \times 10^{-3}$ tonCO_2/L, $E_e = 0.5 \times 10^{-3}$ tonCO_2/kWh, $\times 10^{-3}$ tonCO_2/kWh, and u is uniformly distributed in which $\alpha = 0$ and $\beta = 0.04$, with $E[u] = 0.02$.

In the SSSD policy, the expected total cost (□□□) is \$391,437,636.4/year obtained at □1= 0.01515, □2 = 0.04415, and □ = 0.05930 with Q is 29,740.030 unit. The □□□ produced is 17.47 tonCO_2/year. It will be compared with expected total cost and carbon emission under SSMD policy. Numerical example for SSMD policy is started at n=2 until it shows the convexity. Table 2 shows the result of numerical example for using SSMD policy.

Table 2. Expected total cost and carbon emission for SSMD policy

n	*T* (10^{-5})	*T_1* (10^{-5})	*T_2* (10^{-5})	*ETC*	*ETE*
2	6239	1594	4645	391,302,436.6	17.47
3	6373	1629	4744	391,266,479.0	20.05
4	6458	1650	4808	391,256,109.5	22.37
5*	6523	1667	4856	391,256,071.0	24.61
6	6578	1681	4897	391,261,201.3	26.80

7	6626	1693	4933	391,269,268.0	28.96
8	6670	1705	4966	391,279,148.7	31.09
9	6712	1715	4997	391,290,218.2	33.19
10	6752	1726	5026	391,302,100.0	35.27

According to Table 2, n*=5 is the minimum value of joint expected total cost can be obtained with T_1 = 0.01667, T_2 = 0.04856, and T = 0.06523. The minimum value of joint expected total carbon emissions is 24.61 tonCO_2/year. The optimum Q is 6527.475 units.

6. CONCLUSIONS

This paper is an extension of Daryanto et al. (2019) by considering imperfect quality deteriorating item mixing with VMI systems to consider minimum total cost and carbon emissions that caused from supply chain process between supplier and buyer. Deteriorating items need special maintenance to keep the product life longer that produces significant carbon emissions. Thus, a good inventory management is needed to be able obtaining the goal of enterprise. This study compares two different transportation case, SSSD and SSMD policy, to find the optimum total cost and carbon emission.

From the model development and the numerical example can be found that SSMD policy save more cost comparing to SSSD policy, because it shows convexity form. Also from research finding it can be found optimum number of deliveries per order (n) and the delivery lot size (Q) with the time length of the production and the non-production in each cycle (T, T_1, T_2). This finding can help supply chain managers to take a decision regarding inventory management. Future result might extend this findings to use the vendor managed inventory systems with consignment stocks systems or consider reworking the defective products.

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